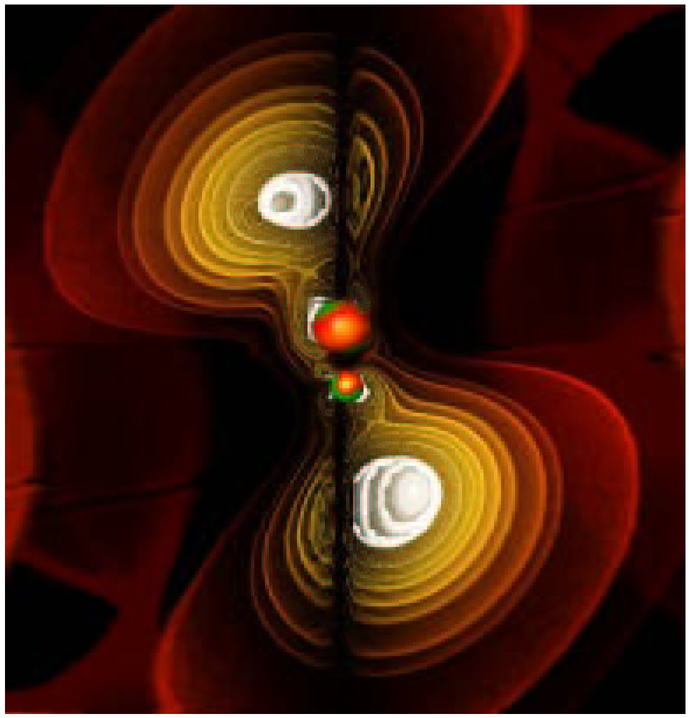
EU NETWORK
THEORETICAL FOUNDATIONS
OF SOURCES FOR GRAVITATIONAL WAVE ASTRONOMY
NEWSLETTER



This is the second issue of the Newsletters of the EU Network *Theoretical Foundations of Sources of Gravitational Wave Astronomy* and it follows the Mid-term Review Meeting held in Southampton, January 31st - February 3rd , 2002. It includes contributions from EU-postdoc and from young people who presented a talk at the meeting, and it illustrates very nicely the activity which is going on and how the collaboration between different groups is evolving and growing.

Contents

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Mini-CV

2000-2003: Ph.D. student in Astrophysics/Numerical Relativity at SISSA/ISAS (International School of Advanced Studies), via Beirut 2/4, 34014 Trieste.

1995-2000: undergraduate studies in Physics, at Università degli Studi di Torino.

The EU hydro code

The newly born (January 2002) relativistic hydrodynamics code for the European Network is an implementation in the Cactus4 Computational Toolkit of the Valencia formulation of the relativistic hydrodynamics equations. Its great advantages are well known: it avoids the need for implicit schemes and for artificial dissipation terms to handle discontinuities and it allows the use of state-of-the-art high resolution shock capturing schemes from classical fluid dynamics. Recent advances in the treatment of spacetime evolution are included in the code (e.g. the implementation in Cactus4 of the ADM_BSSN formulation). An implementation by Ian Hawke of the method of line is used for the simultaneous time update of all differential equations.

The initial version of the code has been written by Ian Hawke, Luca Baiotti and Pedro Montero. Substantial contributions have and will come from Guillaume Faye, Joachim Frieben, José Antonio Font, Tom Goodale, Eric Gourgoulhon, Francisco Guzman, Scott Hawley, Jose Maria Ibanez, Gerd Lanfermann, Luciano Rezzolla, Ed Seidel, Nikolaus Stergioulas...

Description of the code

The first non-technical (e.g. registering variable in Cactus) step performed by the new thorns is of course setting initial data. Up to now we have initial data for shocktube tests and TOV stars. We are inserting the rotating NS initial data (RNSID) routines by N. Stergioulas. For other real runs (binary systems), initial data will also be provided by external thorns, already available in particular from the Meudon group.

Then the source terms in the conservation equation, coming from the curvature of spacetime and depending solely on the metric and on the stress-energy tensor, but not on its derivatives, are computed.

Since hydrodynamical variables are defined in grid centers, before feeding them to the Riemann solver it is necessary to extend them to grid cell boundaries. This is a very important step for the long term stability of the whole code. We have now three slope-limiter methods: minmod, van Leer monotonized centered (MC) and Superbee and the third order piece-wise parabolic method (PPM) by J.A. Font. Possibly, higher order piece-wise polynomial methods, available for Essentially Non-Oscillatory (ENO) schemes will be implemented.

Up to now we have only implemented reconstruction on primitive variables, which is less computationally expensive. However, reconstruction on conserved variables gives better results (because the profile near discontinuities are less sharp, for example). Best of all would probably be reconstructing on characteristic variables.

The cell boundary extended data are passed to the Riemann solvers. These are made of routines solving eigenproblems on the Jacobian matrices at each cell interface. We have already implemented three Riemann solvers: Harten-Lax-van Leer-Einfeldt, Roe and Marquina (by J.A. Font).

Then fluxes and source terms are summed and passed to the MoL thorn, which performs the time evolution simultaneously to all other (spacetime) evolution equations.

Finally, the boundaries are updated.

Improvements

Planned near-future improvements include:

- Inserting explicit formula for the computation of left eigenvectors in Riemann solvers. Actually they are computed through the inversion of the 5x5 characteristic matrix at each grid point and this is very expensive. There was no other way to do it before the explicit formulas for the left eigenvectors were given recently, see for example [1]. This is about to be implemented in the EU code by J. Frieben.
- Finishing rewriting ADM-BSSN to make it work with MoL. Now done by Ian Hawke.
- Including different numerical methods: now Marquina and PPM have been added.
- Including different EOS (also from tables), through the CactusEOS arrangement by T. Goodale.
- Checking coupling of matter with scalar (gravitational) field. In addition to giving rise to new possibly interesting physical results, this coupling could amplify numerical oscillation of the hydro code, if they are there and so be a further, very strict, test for the entire code. A thorn to evolve scalar fields has been written by F. Guzman.
- Using Carpet, the Fixed Mesh Refinement code from Eric Schnetter at Tuebingen University.

And of course future plans include lots of tests. First of all we will reproduce all the results of [2].

Physics

After these and further developments and many more tests, the European Network researchers and students will be able to use this code for carrying out simulations of general relativistic hydrodynamics in various scenarios. The first problem to be addressed will be NS processes in the non-linear regime, since a considerable expertise has been achieved in this area by many members of the Network with previous and independent codes. Eventually, this will also lead to reliable results about gravitational wave signals from merging NS and from all other combinations of binary compact objects. Furthermore, this will build the initial scenario for the high density matter torus expected to form around a rapidly rotating newly collapsed compact object after the coalescence of a binary system. Also in this case we will be able to study the accretion of matter onto the compact object and the emission of gravitational waves that are produced during this process. The evolutions produced by the code will be used as a background to study perturbations to different processes, including perturbations to gravitational collapse.

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2 Emanuele Berti (Thessaloniki)

Mini-CV

- Born: Soriano nel Cimino (VT), Italy, 10-28-1974
- October 30, 1998: **Laurea degree in Theoretical Physics** (Laurea in Fisica) at the University of Rome *La Sapienza*, full marks *cum laude* (**110/110** *e lode*). Supervisors: Valeria Ferrari, Omar Benhar.
- December 1998 - June 1999: contract with the *Istituto dell'Enciclopedia Italiana Treccani* (Italian Encyclopedia) as a scientific reviewer for papers to be published in *Frontiere della Vita* (a monograph on Biology). I took care of a section on neural networks under the direction of Giorgio Parisi and Daniel Amit.
- January 28, 2002: **Ph. D. in Astrophysics** at the University of Rome *La Sapienza*. Supervisor: prof. Valeria Ferrari.
- Post-doctoral position at the Aristotle University of Thessaloniki (local coordinator: prof. K. D. Kokkotas) starting November 11, 2001 within the European Network on *Sources of Gravitational Waves*.

Gravitational waves from Extrasolar Planetary Systems

The emission of gravitational waves from binary systems can be computed using different approximation schemes. The simplest approximation is to assume that the components of the binary are *pointlike* particles following *Newtonian* orbits, and comput the emitted radiation from the time variation of the quadrupole moment of the system, according to the standard quadrupole formula.

A more accurate treatment can be achieved in many different ways. One of the most common approaches considers the members of the binary as point particles, but includes *Post-Newtonian* (PN) corrections to the equations of motion, i.e., basically, corrections in powers of v/c (where v is the orbital velocity). However, taking into account the internal structure of the members of the binary system is, in general, a very difficult task.

Some relevant information on the effects played by stellar structure on the emission of gravitational waves can be gained using *perturbative approaches*. In this case, the key idea is that, when a component of the binary has mass much smaller than the other ($m_0 \ll M$), we can solve *exactly* the Einstein equations coupled to hydrodynamics to obtain the equilibrium structure of the larger body (that we shall assume for simplicity to be non–rotating), and treat the small mass m_0 as inducing perturbations on the (known) geometry of space-time obtained through this procedure. We can solve the perturbation equations inside the star imposing regularity conditions for the perturbations at the center; then we match this interior solution to the solution of the Zerilli (or BPT) equations, which describe perturbations in the exterior vacuum. These equations have, in our case, a source term, coming from the stress-energy tensor of the small, pointlike particle [1, 2].

This procedure works remarkably well for extrasolar planetary systems (EPSs). We shall call EPSs both systems composed of a solar-type star and a planet, and systems composed of a solar-type star and a brown dwarf. The first EPS was discovered in 1992; since then about 80 systems composed of one or more planets orbiting around a main sequence solar-type star have been discovered. Some of these systems are well characterized, since the mass and radius of the central star, the mass of the planets and their orbital parameters can be deduced from observations, and their features are quite surprising when compared to those of planets in the solar system. The most eccentric planet in our solar system is Mercury, with an eccentricity $e = 0.2$, and it is also the planet with the shortest orbital period ($T = 88$ days); about half of the planets in the recently discovered EPSs, on the other hand, have large eccentricities (greater than 0.3), and more than one third of these planets have periods shorter than Mercury (some of them have orbital periods of just a few days!). Furthermore, planets with short orbital periods are typically on almost circular orbits.

Using the available experimental data on the orbital features of various observed EPSs, we have been able to give a first estimate of the amplitudes and frequencies of the gravitational radiation they emit [3]. In particular, one of the observed EPSs, HD283750, emits waves with a maximum characteristic amplitude $h_c \sim 4 \cdot 10^{-23}$ and a corresponding frequency $\nu \sim 1.3 \cdot 10^{-5}$ Hz. These values are quite interesting, especially when compared to the maximum amplitude $h_c \sim 10^{-23}$ and frequency $\nu \sim 1.4 \cdot 10^{-4}$ Hz of the Hulse-Taylor binary pulsar (PSR 1913+16). The large value of the amplitude is due to the fact that observed EPSs are typically quite close to Earth (few tens of parsecs, to be compared with the distance $d \sim 5$ kpc of PSR 1913+16).

Then we turned our attention on an interesting mechanism of gravitational wave emission, not so directly related to the orbital motion of the system. It is known from the theory of stellar pulsations that a star, when perturbed from its equilibrium configuration, oscillates in a set of so-called *quasi-normal modes*. These modes have complex frequencies; the imaginary part corresponds to a damping of the mechanical energy of oscillation, due to the emission of gravitational waves. An interesting feature of the stellar quasi-normal modes is that they can be **excited** by tidal interactions with the orbiting planet; this happens whenever the Keplerian frequency of the orbiting planet ω_k is related to the frequency of some stellar mode ω_{mode} by the relation $\omega_{mode} = 2\omega_k$.

Motivated by observations, we have considered the possibility for a planet to get sufficiently close to the central star to excite the lowest order g-modes, and possibly the fundamental one, without being disrupted by tidal forces or melted by the central star. We have shown that temperature effects are typically negligible with respect to tidal effects. Studying tidal effects through a Roche-lobe analysis we have shown that: 1) for solar type stars, the resonant excitation of the low-order g-modes is in principle possible; 2) EPSs at a fiducial distance $D = 10$ pc can emit, just because of their orbital motion, waves of amplitude as large as $h_c^{max} \sim 10^{-22}$ and frequency as high as $\nu^{max} \sim 10^{-4}$ Hz without being torn apart by tidal effects.

When the planet is close enough to a resonance, an integration of the perturbed Einstein equations shows that the power lost by the system in gravitational waves is larger due to the tidal excitation of the stellar modes, and the orbit tends to shrink more rapidly. Including gravitational radiation reaction effects on the planetary orbit tells us how fast the orbit shrinks: in this way, we can obtain typical timescales for a planet to get off-resonance. We have performed a radiation reaction analysis [4] showing that, for the solar-like stellar model we have chosen, a brown dwarf exciting the mode g_4 could reach amplitudes $\sim 2 \cdot 10^{-20}$ for ~ 3 years, and $\sim 4 \cdot 10^{-21}$ for ~ 400 years; on the other hand, a planet like Jupiter resonant with the mode g_{10} could reach amplitudes $\sim 3 \cdot 10^{-22}$ for ~ 2 years, and $\sim 6 \cdot 10^{-23}$ for ~ 300 years.

Figure 1 summarizes our results. There we show the sensitivity curve for LISA (i.e., the amplitude required at each frequency to observe a signal with signal-to-noise ratio greater than 5) along with some of the most interesting results obtained by our analysis. Though significantly larger in amplitude than the signal emitted by the binary pulsar, the waves emitted by the *observed* system HD283750 are not detectable with the current LISA design.

The same is true for a "resonant" system composed by a solar-type star and a Jupiter-like planet, but *a brown dwarf in resonant conditions could potentially be detectable.*

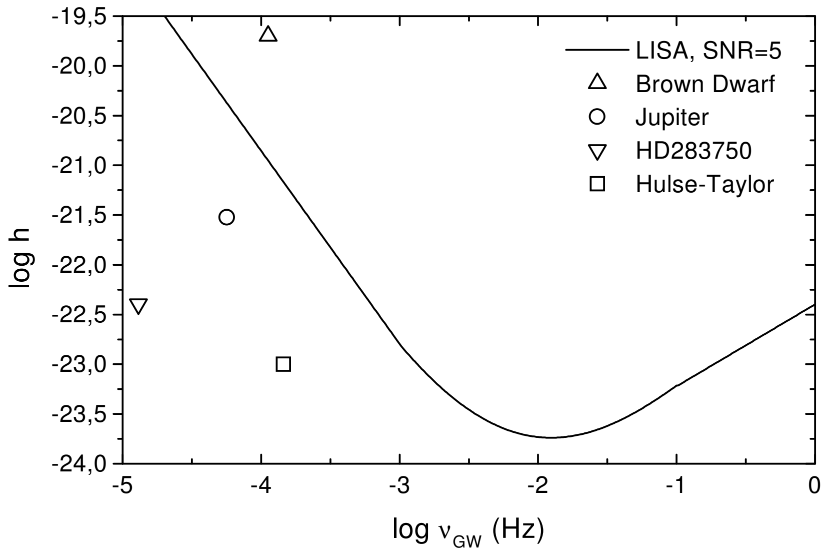

Figure 1: The solid curve is LISA's sensitivity curve, representing the amplitude required to observe a signal with signal-to-noise ratio greater than 5. The different symbols represent a brown dwarf in resonant conditions with amplification 50, a Jupiter-like planet in resonant conditions with the same amplification factor, and two *observed* binaries: HD283750 (an extrasolar planetary system) and PSR 1913+16 (the Hulse-Taylor binary pulsar).

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3 Peter Diener (AEI)

Mini-CV

- Name: Peter Diener
- August 2000 - present: EU-post-doc, Astrophysical Relativity Group, AEI Potsdam.
- February 2000 – August 2000: Post-doc, Tapir Group, Caltech Pasadena
- November 1997 - December 1999: Post-doc, Astronomy department, University of Texas at Austin.
- 1994 - 1997: PhD., *Tidal Interaction of Stars with Massive Black Holes*. Supervisor Igor Novikov, University of Copenhagen

Post Newtonian and Kerr-Schild BBH initial data

We currently have groups working on implementing several different kinds of initial data for Binary Black Holes (BBHs). The first kind is based on Post-Newtonian expressions for the metric and extrinsic curvature up to 2.5 PN in the ADM gauge provided by Jaranowski and Schäfer in [1]. These expressions are then used directly in the standard York-Lichnerowicz conformal decomposition in order to solve the constraints as four coupled elliptic equations. This project was started by Bernd Brügmann, Manuela Campanelli and me, but most of the work implementing it have been done by Wolfgang Tichy as reported by him in Southampton. We are currently at the stage where we are able to solve the constraint equations so now the fun of studying the physics starts.

The second kind is based on superposition of two black hole metrics in Kerr-Schild form proposed in [2]. One black hole in Kerr-Schild form is given by

$$g_{\mu\nu} = \eta_{\mu\nu} + 2Hl_\mu l\nu, \tag{1}$$

where $\eta_{\mu\nu}$ is the Minkowskian, H is a scalar function and l_μ is an ingoing null vector. For a Schwarzschild black hole we have

$$H = M/r, \quad l_\mu = \left(1, \frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right), \tag{2}$$

where $r = \sqrt{x^2 + y^2 + z^2}$. From equation (1) it can be seen that the description of the black hole is contained entirely in the second term. The idea now is to make a binary black hole metric by superposing two such terms, describing two separate black holes

$$g_{\mu\nu} = \eta_{\mu\nu} + 2\,{}_1H\,{}_1l_\mu\,{}_1l_\nu + 2\,{}_2H\,{}_2l_\mu\,{}_2l_\nu. \tag{3}$$

Of course this metric does not satisfy the constraint equations and in fact the constraints diverge at the 2 black hole singularities. From physical grounds, however, we expect that the influence of the second black hole should be unimportant near the singularity of the first black hole. This property is not satisfied by the simple ansatz of superposing the two black holes. This can be remedied by multiplying each black hole term with an attenuation function (${}_1B$ and ${}_2B$) that vanishes near the second black hole, forcing the resulting metric to be a solution in the neighbourhood of the singularities. The final ansatz for a binary black hole metric is then

$$g_{\mu\nu} = \eta_{\mu\nu} + 2\,{}_1B\,{}_1H\,{}_1l_\mu\,{}_1l_\nu + 2\,{}_2B\,{}_2H\,{}_2l_\mu\,{}_2l_\nu. \tag{4}$$

This metric is then used to extract a 3-metric and extrinsic curvature to be used as the conformal metric and extrinsic curvature in the York-Lichnerowicz conformal decomposition in order to finally obtain a solution to the constraint equations describing a BBH system.

This kind of data are pursued by Denis Pollney and Michael Koppitz within the framework of Cactus and BAM and by myself within the framework of an Adaptive Mesh Refinement (AMR) package called FTT. At present I'm debugging my Gauss-Seidel relaxation routine, which are an important part of a non-linear multigrid scheme as a smoother. The multigrid scheme should be in place, so as soon as I have the relaxation routine working, I should be able to make rapid progress.

Binary black hole evolutions

One of the main projects of the group at the AEI is of course the numerical evolution of black hole space times. Recent developments in formulating new gauge conditions have allowed us to extend the lifetime of the numerical simulations long enough to be able to extract interesting physical information.

For the initial data we are mostly using Brandt-Brügmann puncture initial data. However since the parameter space is very big, we are currently limiting ourselves to the study of a few one parameter families of initial data. These families are based on the estimation of the Innermost Stable Circular Orbit (ISCO) in [3]. One of these is the P-sequence, where the black holes are positioned at the ISCO separation along the y-axis, but with varying linear momentum in the x-direction from zero (head on collision) to the ISCO momentum (circular orbit). The second is the PI sequence where the separation and momentum are both changed to maintain circular orbits, going from PI0 corresponding to the ISCO with a separation of $4.9M$ to PI10 with a separation of $14M$.

At present we are mainly using the so called hyperbolic K-driver condition for the lapse and the hyperbolic Gamma-driver condition for the shift. I will not go into detail about these conditions right now. Suffice to say, that they come in several different flavours with even more control parameters. Some parameter values are working well with some systems, while other parameter values are working well with other systems. For that reason we have to do extensive parameter searches to find the optimal parameters for a given system.

Especially for the PI sequence, it became clear that in order to have long numerical evolutions, the above mentioned shift conditions were not adequate, because of distortions due to the rotational motion of the system. It was found that by introducing an initial co-rotation shift and then let the standard shift conditions take over, it was possible to extend the lifetime. If done right it should be possible to effectively do the evolution within a co-rotating frame where it would look like a slow head on collision. However the angular velocity does not stay constant, so a way to modify the rotational part of the shift was implemented, based on keeping track of the drift of the centroid of the Apparent Horizon (AH) and adjusting the shift to compensate. This we call drift correction. Unfortunately finding the AH is rather slow, so we can not afford to find it every time step. This we would like to do in order for the drift correction to be as smooth as possible. For that reason Scott Hawley and Denis Pollney are currently investigating the possibility of doing the drift correction based on the centroid of the region where the lapse has collapsed.

Another crucial ingredient in long term evolution is the excision of the region inside the event horizon. The idea, of course, is that you can remove all the nasty stuff going on near the singularity without fear of this affecting the evolution anything outside of the horizon. This is another active research area at the AEI.

Using the gauge conditions, co-rotation shift, drift correction and excision Denis Pollney, Frank Herrmann and Thomas Radke was able to do a long term evolution ($T > 80M$) of a PI3 system (initial separation of $6.5M$) producing 3D data to be used for a movie, that are going to be broadcast on the Discovery Channel in June (a show about astrophysical processes in the Universe). The main problem during this run, was not to keep the code from crashing because of the physics, but to keep it from crashing because of technical things like hard disks failing, running out of inodes thereby losing data or the machine going down. However that was enough to make them keep long hours for a significant stretch of time.

By improving the gauge and the boundary conditions we hope soon to be able to evolve a BBH long enough to do a full orbit.

My main involvement in the BBH runs has been the study of the head on collision of 2 black holes from the ISCO separation, which we call BBH0, though I have also been involved in parameter searches for PI runs. However the BBH0 system is benign enough that you really do not need excision at all. With just the basic gauge conditions and very importantly the right parameters to go along with them, we can essentially evolve this system forever. This is true for our K-driver lapse condition as well as maximal slicing (as I showed at the Southampton meeting). However, maximal gets trickier at higher resolution. Due to sharp features in the metric and extrinsic curvature it can happen that a multigrid elliptic solver will diverge, while a standard relaxation scheme will converge. Unfortunately a standard relaxation scheme is orders of magnitude slower than multigrid making it necessary to explore other options, which I hope to do soon.

As a result of my work, figure 2 shows the Zerilli $l = 2$, $m = 0$ waveform extracted from BBH0 runs at 3 different resolutions. The phase is slightly different at the different

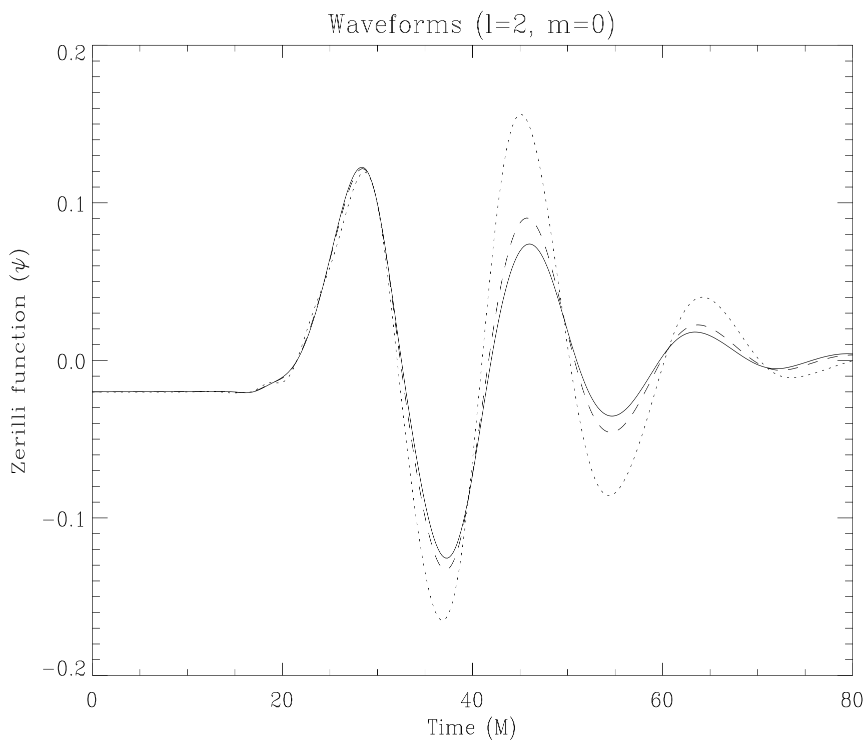

Figure 2: Zerilli $l = 2$, $m = 0$ waveforms extracted from BBH0 runs at 3 different resolutions. Solid line : $0.032M$, dashed line : $0.064M$ and dotted line : $0.128M$

resolutions but still the amplitude shows very nice second order convergence until about $T = 80M$.

Other projects at the AEI

In order not to give the impression that BBH initial data and evolution is the only thing going on at the AEI, I convinced some of my fellow young researchers to write a few paragraphs about parts of their work. So in the following four sections: Nils Dorband will write about his work on Fixed Mesh Refinement (FMR) using Carpet, Francisco Guzman will write about his work on scalar fields, Ian Hawke will illustrate the Method of Lines (MoL) which he implemented in Cactus, and Michael Koppitz will describe the first numerical evolutions of the Meudon initial data for BBHs.

FMR

To get ready for the serious use of adaptive mesh refinement (AMR) in numerical relativity it is important to understand the possibilities and problems of fixed mesh refinement (FMR). Apart from regridding algorithms like a clusterer in AMR, both methods are very similar and should show the same numerical error and convergence behaviour.

We are using Carpet, a driver for Cactus written by Erik Schnetter (U. Tuebingen), to apply FMR for some simple test cases, such as the 3D wave equation. We have only been finding first-order convergence in our Carpet runs using initial data of the form $\phi = \frac{1}{r}$, $r = \sqrt{x^2 + y^2 + z^2}$, which is contrary to our expectation based on runs with simple standalone C and Mathematica codes. As soon as we understand this loss of one order of convergence we will move on to more complicated evolution schemes like ADM or ADM-BSSN.

Scalar fields

Various members of our group (Dramlitsch, Guzman, Hawley, Seidel) have over the years done significant work on self-gravitating scalar fields, both in spherical symmetry and in full 3D. As scalar fields are thought to play an important role in the Universe, and through a well known Jeans instability, a scalar field background can fragment and collapse to form stable bosonic objects (Boson Stars, or BS) or black holes, such objects may actually be abundant in the Universe. Since they have very different internal structure and sizes from astrophysical objects such as black holes, neutron stars, or even quark stars, interactions of boson stars with their environment may produce unique gravitational wave signals, which we intend to investigate.

We have recently developed a completely new thorn to evolve the scalar fields, and using the generalized matter-spacetime interfaces developed for Cactus, the GR Hydrodynamics and scalar field thorns will be completely interchangeable as matter sources for coupled spacetime-matter evolutions.

The scalar field thorn provides a variety of evolution schemes to work with, and therefore it will be a simple consequence that fundamental differences between distinct evolution schemes should be found in 3D as it is known in 1D for long lived solitonic solutions; in this way the scalar field will act once again as a testbed for numerical techniques.

Nevertheless, scalar fields apart of being testbeds, they provide astrophysical and cosmological implications besides those concerning wave extraction, since the roles played by scalar fields in the cosmological models are diverse: the GUTs and the topological defects possibly formed at that epoch, which are nothing but combinations of scalar fields with certain symmetries that now are being generalized with our code; the quintessential models of dark matter, which have proved to be successful when it is assumed spherical symmetry and cosmologically motivated scalar field potentials, and thus in 3D it is now possible to study the hierarchical structure formation under such ideas. Branches all of these, that are currently under our research.

Method of Lines

A simple way of approaching a complex physics problem, such as the calculation of the gravitational wave signature of a coalescing neutron star binary, is to split the problem into small pieces which can be tackled individually. In practice this means considering many different systems of differential equations, for example the 3+1 equations for evolving the spacetime and the equations for evolving the hydrodynamics. Highly accurate methods can be applied to the simpler subproblems both for testing and for extracting interesting physics.

However, it is not guaranteed that the stability and accuracy of these methods will be retained when the different models are coupled together. The diagrams in [4] indicate the problems inherent in retaining high order convergence. With the inclusion of more models such as scalar fields, electrodynamics or radiation transport the problems at the numerical level increase rapidly.

The Method of Lines (MoL) is a framework for solving a coupled system of differential equations that avoids this coupling problem. This framework is illustrated by considering a simple one dimensional inhomogeneous conservation law

$$\partial_t \mathbf{q} + \partial_x \mathbf{f}(\mathbf{q}) = \mathbf{s}(\mathbf{q}), \tag{5}$$

although it should be emphasized that the MoL framework can be applied to any system of partial differential equations. We assume the existence of a computational grid and discretize in space, finding

$$\partial_t \mathbf{q} + \frac{\mathbf{f}(\mathbf{q}_{i+1/2}) - \mathbf{f}(\mathbf{q}_{i-1/2})}{x_{i+1/2} - x_{i-1/2}} = \mathbf{s}(\mathbf{q}_i) \tag{6}$$

using the notation that x_i refers to the centre of a computational cell and $x_{i\pm1/2}$ to the boundaries of a computational cell.

A trivial rearrangement of this equation gives a system of ordinary differential equations which can be written in the form

$$\partial_t \mathbf{q} = \mathbf{L}(\mathbf{q}), \tag{7}$$

where $\mathbf{L}$ is a discrete spatial operator. This system of ordinary differential equations can be solved with any standard numerical method, given a method of calculating $\mathbf{L}$. If another model is to be coupled into the system the same transformation is performed and the same numerical solver used. In this way the coupling at the numerical level is guaranteed to give the required numerical order of accuracy.

Many existing codes implicitly use the Method of Lines, including the Cactus implementations of the ADM_BSSN system of equations [6] for evolving the spacetime, and the full relativistic hydrodynamics code [4, 5]. A piece of Cactus code has been written that implements the Method of Lines in a general form. This allows the simple coupling of different Cactus thorns at the required order of accuracy provided these thorns can calculate the spatial operator $\mathbf{L}$. This code has been used as the basis of the new hydrodynamics code, as described by Luca Baiotti at the Southampton Network meeting. The implementation of the ADM_BSSN system has been altered to use this generic code as well. When the new hydrodynamics code is used for fully dynamical evolutions we can now guarantee the correct order of convergence, even if we implement higher order numerical methods. Equally, it is particularly simple to implement higher order methods using this framework, and some are already in place.

Meudon Data in Cactus

In [7] and [8] Philippe Grandclément, Eric Gourgoulhon and Silvano Bonazzola introduced a solution to the Einstein equations describing a binary black hole system. Using the simplifying assumptions that the black holes stay in circular orbit and that the spacetime is conformally flat, the remaining five equations to solve were the ones known from the Thin Sandwich ansatz for the initial value problem. They solved the equations using spectral methods.

The idea now is to take the data the group in Meudon provided and evolve it using Cactus. Therefore they provided us with some files containing the coefficients of the spectral decomposition and with a tool to put the functions on the numerical grid of Cactus.

To use the data in Cactus it was necessary to fill in the data inside the throats. That was done by the isometry condition used to construct the data. Now these data are fully parallelized read into Cactus who's machinery can be used to evolve it and extract all the wanted information.

For now there is only a short period of 3 M of evolution done before the code crashed but there is plenty of possibilities to extend this running time. The extracted data agree with the expectations in that the horizons found are almost identical to the throats and the error (of the momentum constraint) introduced by the method of solving the equations is shown to be very small.

We are very confident that we will soon be able to evolve this new set of initial data and extract physical data which will enable us to compare it with evolutions of other sets with respect to it's physical meaning.

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4 Guillaume Faye (Jena)

Mini-CV

Personal data

Date of Birth: July 3rd 1973. Nationality: French.

Background

1995-1996: "D.E.A." of Theoretical Physics (i.e. postgraduate certificate), Doctoral School of Physics of Paris and its Vicinity, University of Paris-Sud XI (France).

1993-1996: "Magistère" of Physics of Orsay University (i.e. high-level University degree awarded at the end of an engineering-oriented academic education), University of Paris-Sud XI (France).

1996-1999: PhD thesis in theoretical physics, University of Paris-Sud XI (France).

Position held and research activities

1996-1999: PhD student at the Paris-Meudon Observatory (**OP** knot, France).
Supervisor: L. Blanchet.
Subject: Equations of motion of a compact binary at the 3rd post-Newtonian order (computation of elementary potentials).

1999-2000: Scientist appointed by the Army to the French National Agency for Aerospace Research (ONERA, France).
Subject: Propagation of light in a turbulent atmosphere beyond the Rytov model (theory and numerical simulations).

2000-2002: Post-doctoral position in the European Network at the Institute of Theoretical Physics, Jena (Germany).
Subject: Approximation methods in gravitodynamics

On a global expression for the 3-metric of a black-hole binary at the 2nd post-Newtonian order

There appear two major difficulties when solving numerically the wave-like equations describing the field dynamics of gravitational-wave sources. (1) Numerical computations start in practice at finite time, whereas, in most cases the boundary conditions are given at past null infinity. (2) The space sections cannot be compactified by spherical inversion without inducing spurious reflections. As a first consequence, the Cauchy formulation, which entirely relies on the initial values of the field and their time derivatives, reveals to be unadapted. On the other hand, in the case where boundary conditions are imposed at each step of time, the boundary domains are restricted to having finite size. This means for isolated systems that the Kirchhoff-Sommerfeld conditions must be re-stated at a finite distance from the origin (see recent work of J. Novak from the Meudon group). But if an approximate solution is known for a particular system, it may be used to improve the accuracy of the field computed on the boundary domain.

In this context, the **FSU** knot obtained an approximate expression for a part of the spatial 3-metric of a black-hole binary, that reduces to the 2nd post-Newtonian order (2PN *i.e.* $1/c^4$) in the near zone, but remains a good approximation over the entire space in the Arnowitt-Deser-Misner (ADM) gauge (equivalent to the gauge used by the Meudon group at this order). The black-holes are actually modelled by two Dirac functions representing point particles, and the divergences resulting from the self-interaction are removed by means of Hadamard regularization. The conformally flat part $\psi^4\delta_{ij}$ of the 3-metric g_{ij} can simply be taken as equal to its well-known 2PN expression (see *e.g.* [1]). There remains the transverse trace-free part $h_{ij}^{\rm TT} = g_{ij} - \psi^4\delta_{ij}$ which is responsible for the radiation. In a post-Minkowskian scheme, this term satisfies a wave-like equation and $h_{ij}^{\rm TT}$ is formally given by a retarded integral. In the present approximation, the source integrand is further simplified by dropping all contributions beyond the leading 2PN order (equation 26 in [2]). This approximation is valid in the near zone as well as in the far zone. Now, the source naturally splits up into a kinetic and a potential part at the leading 2PN order. The retarded integral of the kinetic part was essentially written as a sum of simple integrals depending only, at a given time and field point, on the particle positions or momenta. The latter integrals were explicitly evaluated in the case where the gravitational field is weak enough, *i.e.* when the velocities can be considered as constant at each step of time integration. The remaining potential part is much more difficult to treat. Calculations are in progress.

First correction to the Wilson approximation

In numerical relativity, the computation of the gravitational field can be greatly simplified by resorting to the Wilson approximation in which the 3-metric is assumed to be conformally flat. The approximate solution is actually exact in the case of spherical symmetry and, moreover, it includes the 1PN order. This can be easily seen in the ADM gauge where the non-conformally flat part of the 3-metric is the transverse trace-free matrix $h_{ij}^{\rm TT} = \mathcal{O}(1/c^4)$. For an isolated perfect fluid with an arbitrary equation of state, $h_{ij}^{\rm TT}$, at the leading order, obeys a Poisson equation, but the source depends on several iterated Poisson integrals produced by the transverse trace-free projection. An alternative equation suitable for numerical treatments was derived. The new source depends only on the Newtonian potential in addition to the matter variables, namely the baryonic density, the fluid momentum and the fluid entropy. Only two terms of twenty-four entering the source have non-compact support. As they converge faster than $1/r^4$, r being the distance from the origin, they can be integrated over the whole space by spherical inversion. The first correction to the lapse function is given by a Poisson integral having the same type of quickly converging source. As for the shift function, it is not affected at this order (2PN).

Highly accurate calculation of rotating neutron stars

The study of relativistic, axisymmetric and stationary, uniformly rotating perfect fluid bodies is motivated by extraordinarily compact astrophysical objects like neutron stars. Several numerical codes have been developed in order to calculate the structure and the gravitational field of these bodies. While they obtain an accuracy of up to 5 digits for sufficiently smooth equations of state, these methods yield fewer than 4 digits in the case of stiff equations of state (e.g., for constant density), which is due to particular Gibbs phenomena at the star's surface. In order to avoid these Gibbs phenomena, Bonazzola et al. (1998) used a multi-domain spectral method with which they were able to achieve an accuracy of 12 digits for the Maclaurin sequence of homogeneous Newtonian bodies.

In recent work [3], the **FSU** knot introduced a new numerical code, which is based on a multi-domain spectral method for representing all metric functions. This method was intending to be used for investigating neutron stars with realistic equations of state. In particular, the multi-domain method lends itself to considering several layers inside the star, which are characterized by different equations of state. This method yields a hitherto unobtainable accuracy which permits its application even for extremely flattened Einsteinian bodies and in the mass-shedding limit.

In the method proposed here, the interior and the exterior of the star are separately mapped onto squares. The corresponding coordinate transformation involves the unknown shape of the star. In consideration of the relevant boundary and transition conditions, all metric functions are represented by two-dimensional Chebyshev-expansions with respect to the coordinates defined on the squares. Then, the differential equations as well as the remaining transition conditions are used to determine the Chebyshev coefficients by means of a Newton-Raphson method.

Figure 3 shows an example of highly flattened Einsteinian bodies (in meridional cross-section) obtained by using these methods. The relative error is less than 10^{-6} for typical physical quantities such as the angular momentum, gravitational mass and relativistic red-shift for these bodies.

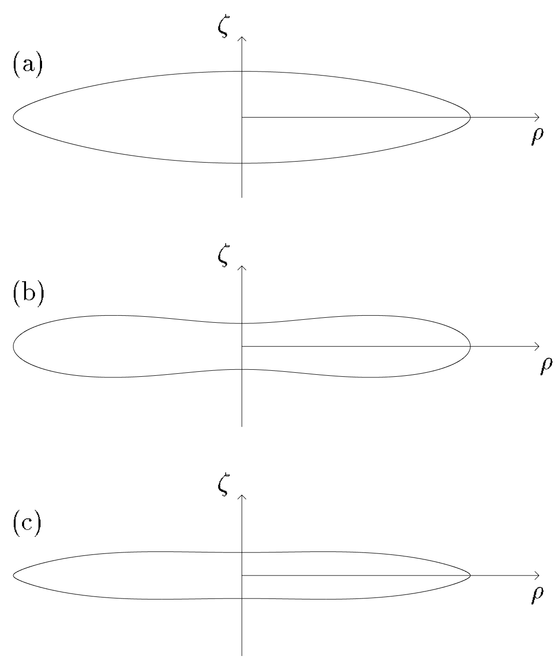

Figure 3: Meridional cross-sections of three highly flattened constant density models (see main text)

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5 Joachim Frieben (Valencia)

Personal Information

I obtained my master degree in Physics in 1992 from the University of Heidelberg in Germany. During my master thesis in the framework of the 'CERES' (NA45) experiment at **CERN**, I built a prototype electron-positron trigger detector for later integration into the experimental setup [1].

The following year, I moved to Paris to carry out my graduate studies in the group of Silvano Bonazzola in **Meudon (OP)**. My research was focused on the application of Spectral Methods to 3-D Numerical Relativity and the investigation of the viscosity triggered secular instability in rapidly rotating relativistic stars [2, 3, 4, 5].

After working in private business from 1998 to 2000, I joined Ed Seidel's group at the **AEI (Golm)** as a visitor in February 2001 before being appointed to the network at the beginning of June 2001 as a member of the **Valencia (UVEG)** group. I stayed in Golm until September 2001, where I attended in particular the CACTUS workshop throughout the whole month of July 2001 before moving to Valencia by mid-September 2001.

The OP/UVEG *'Mariage des Maillages'* project

The Valencia group (UVEG) and the Meudon network node (OP) have started a collaboration with the aim of co-developing a 3-D Numerical Relativity code. The main purpose is to evolve dynamical space-times containing some auto-gravitating isolated matter distribution.

Since we are interested in scenarios such as stellar collapse or neutron star binary coalescence, the code's ability to handle shocks appearing in the hydrodynamical quantities is of crucial importance. An important feature is the capability to neatly capture gravitational waves escaping to infinity which implies that a particular attention has to be paid to a proper treatment of boundary conditions.

In order to meet the various and eventually concurrent requirements, we have retained a hybrid approach combining *'High Resolution Shock Capturing'* (HRSC) methods for solving the relativistic hydrodynamical equations and *'(Pseudo)-Spectral'* methods for solving the equations of relativistic gravity. According to the star-like nature of the objects under investigation, we adopt spherical-like coordinates but Cartesian components in order to have to deal with *scalar* quantities only. Coordinate singularities are handled very naturally in the framework of spectral methods. A spectral expansion of the various quantities according to their inherent regularity properties ensures a seamless computation of derived quantities like, e.g., the covariant derivative. As a thumb rule, one can keep in mind that regular operators applied to regular quantities yield regular quantities again. Thus, as opposed to FD methods, spherical-like coordinates being natural coordinates for problems involving star-like objects can be adopted without any concern. Spherical methods are particularly well suited for solving PDEs when the involved functions are smooth. Roughly speaking, this can be assumed to hold for the metric potentials, whereas this assumption does certainly not apply to the hydrodynamical quantities themselves.

We prefer gauge conditions that yield elliptic equations for the gauge variables. They give rise to well-posed boundary problems for which a sound mathematical basis exists. Furthermore, their solution by means of spectral methods is extremely efficient compared to FD based elliptic solvers.

The computational framework for the spectral method part is provided by LORENE, a C++ class library for Numerical Relativity developed by E. Gourgoulhon et al. (OP) which has been applied successfully to various problems of Numerical Relativity in the past. Apart from their usefulness for solving elliptic equations appearing naturally in problems involving quasi-equilibrium configurations, they have been recently applied to the time-dependent classical scalar wave equation where a novel approach for treating the outgoing wave boundary condition has been developed and successfully tested [6]. Once more, the mathematical procedure greatly benefits from the use of spherical coordinates.

The hydrodynamical scheme relies on the work of the Valencia group (UVEG) who has made HRSC schemes become the method of choice for simulations of relativistic astrophysical flows. In our case, the so-called 'TONIK' code, jointly developed by J. A. Font & N. Stergioulas, serves as starting point for the hydrodynamical part of the hybrid code under development [7]. As a consequence, our approach is currently restricted to axisymmetric problems although this limitation shall be removed at a later stage.

In a first step, the original Fortran code has been translated to C/C++ in order to allow for a stepwise introduction of objects and methods from the LORENE class library. At present, LORENE already provides the required functionality for solving the elliptic equations obtained for Wilson's approximation where a conformally flat metric is assumed. It is hence possible to extend the TONIK code which is originally based on the Cowling approximation of a fixed background space-time to some Post-Cowling approximation where the lapse function as the dominant part of the gravitational field is treated as a three-dimensional quantity. Although restricted to the perturbative regime, this approach can already yield interesting insights when applied to the problem of stellar oscillations and instabilities in comparison with the original Cowling approximation. This extension is work in progress. A successive refinement of the approximate scheme will then lead to the well known Wilson approach characterized by the *'conformally flat condition'*. An intermediate step is the investigation of stellar collapse in this approximation, before going beyond this very popular scheme by solving the dynamical field equations, too. Entering the realm of truly dynamical space-times, the investigation of the gravitational wave emission associated with the above-mentioned scenarios is of course the ultimate goal of this project.

Development of CACTUS

Hydrodynamical codes which make use of HRSC schemes require for the most popular ones the knowledge of the full spectral decomposition of the associated Jacobian matrix: of eigenvalues, and of left and right eigenvectors. They are in particular required for the computation of the numerical fluxes across the cell boundaries. The reduced set of eigenvalues and right eigenvectors for the general relativistic hydrodynamical equations had been presented in [8]. Due to the absence of analytical expressions for the left eigenvectors, these are normally computed by numerical inversion of the matrix of right eigenvectors. Unfortunately, this procedure is not only expensive from the computational point of view but also prone for introducing further numerical errors into the numerical solution because of the huge number of arithmetical operations involved.

Recently, a full set of left eigenvectors had been derived for the special relativistic case [9]. It allowed to achieve an impressive performance boost in multi-dimensional applications of special relativistic hydrodynamics when combined with some optimized flux formula that makes explicit use of the left *and* right eigenvectors in the computation of the numerical viscosity matrix [10]. Analytical expressions for the left eigenvectors in the general relativistic case had previously been given in [11]. Recently, J. A. Pons from **Rome (URLSDF)** has been able to derive the flux formula for the general relativistic case specialized to the case of the Roe solver.

The implementation of the analytical expressions for the left eigenvectors and of the optimized formula for the numerical viscosity matrix in the original CACTUS EU-Astro-Hydro thorn allowed to reduce the execution time by 2/3 in the corresponding subroutine where both improvements contributed equally. Given an approximate 15% share of the total execution time spent in the subroutine for different test bed cases, the effective speed-up factor of 3 translates into a reduction of the execution time of the whole code of 10% which has to be compared to the theoretical limit of 15%. The implementation of the optimized viscosity flux vector calculation has also been implemented in the new 'Whisky' CACTUS hydro thorn. A future direction of this work is the extension to other schemes beyond the Roe solver, in particular to the very popular Marquina flux formula.

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6 Ian Jones (Southampton)

I am now in my final year of a post-doc sponsored by the British government. Nevertheless, my field of research is very closely related to that of the Network, and so I've written a brief description of the main area of overlap. I'd like to add that although I will start a post-doc in Penn State this coming September, I hope to continue collaborating with people within the network while I'm abroad.

My main research project is a numerical one: I have written a code to time evolve the linearised perturbation equations of a rotating Newtonian star. The unperturbed background is simply the stationary axisymmetric one appropriate to the star's rotation rate and central density, and is calculated in a separate numerical code. An arbitrary perturbation is then superimposed on this background and evolved using 2-step Lax-Wendroff finite differencing. To ensure that the code runs reasonably rapidly on a standard PC, all perturbations are decomposed into the basis $e^{im\phi}$, and the perturbation in each separate m is evolved separately. The code is therefore two-dimensional in space.

The motivation behind this work is to provide a test-bed for new ideas and formulations before they are incorporated into full-scale three-dimensional hydro codes. In particular, we in Southampton are interested in implementing a local gravitational radiation reaction force, to observe the growth of a CFS-unstable mode, including both mass and current multipole contributions.

This has naturally created two collaborations within the network. Firstly, I have visited the Meudon group several times to learn about the LORENE package, which I am now using to solve the Poisson-type equations which inevitably arise in calculating the radiation reaction force. I hope that from this point on I will be able to develop my code without too much assistance, emailing the Parisians when I get stuck. LORENE is written in C++, a language which I have not encountered before, so much of my time has been spent trying to understand unfamiliar coding techniques.

Secondly, the derivation of the radiation reaction formulae is a problem in post-Newtonian theory. The Jena group are the experts in this, and so once I have mastered the issues involved in numerically solving Poisson-like equations, I will work with Guillaume Faye in implementing the full radiation reaction equations. I hope to begin this phase of the project very soon.

7 Giovanni Miniutti (Rome)

Mini-CV

Final year PhD student at Dipartimento di Fisica, Univ., di Roma "La Sapienza". Research project on *Stellar Perturbations in General Relativity and Emission of Gravitational Waves*, to be completed in October 2002 in the group led by Prof. Valeria Ferrari

Degree in Theoretical Physics (Nov. 25 1999) at the Univ. of Roma "La Sapienza", 110/110 cum laude

Research experience in General Relativity, Gravitational Waves, Physics of Black Holes and Neutron Stars, Relativistic Stellar Perturbations and Oscillations, Spacetime Splitting techniques. Working experience in Fortran programming and symbolic computation software as Maple and Mathematica

Density discontinuities in neutron stars

The composition of a neutron star is known to vary with radius from the center to the surface of the star. The characteristic radial chemical profile is dictated by the history of the star because shell burning, flash nuclear burning and accretion phenomena have leaved layers of different composition near the surface of the star. At the interfaces between different layers, the chemical composition changes abruptly. If no significant diffusion is present, variations in chemical composition appear as density discontinuities at the interfaces. The presence of a discontinuity introduces an additional local source of buoyancy which give rise to discontinuity g-modes even if no other sources of buoyancy are present, i.e. even in zero temperature neutron stars. Density discontinuities in neutron stars are not necessarily confined in the outer layers of the star [1]. They could also arise due to phase transitions in the high density regions of neutron stars, where the equation of state is still poorly known. Several first or second order phase transitions have been proposed and they typically involve pion and/or kaon condensation as well as the transition from ordinary nuclear matter to quark matter. Here we focus on the signatures that might be expected in the gravitational radiation emitted by perturbed neutron stars due to the possible excitation of non-radial discontinuity g-modes.

In order to study discontinuity g-modes in neutron stars, we consider a simple model with a polytropic equation of state [2] of the form

$$p = \begin{cases} K\rho^{\Gamma} & \rho > \rho_{\mathrm{d}} + \Delta\rho \\ K\left(1 + \dfrac{\Delta\rho}{\rho_{\mathrm{d}}}\right)^{\Gamma}\rho^{\Gamma} & \rho < \rho_{\mathrm{d}} \end{cases} \quad , \tag{8}$$

which enables us to insert density discontinuities with amplitude $\Delta\rho$ at density ρ_{d}. We integrate the equations of stellar perturbations in General Relativity using the usual boundary conditions:

- regularity of the perturbation variables at the center of the star;
- vanishing of the pressure and of the Lagrangian pressure perturbation at the surface of the star;
- outgoing wave condition at infinity.

In addition, as far as density discontinuities are concerned, we add additional matching conditions at the interface where the density changes abruptly. We require the continuity of the radial displacement ξ^r (due to the continuity equation) and of the Lagrangian pressure perturbation Δp (in order to avoid infinite acceleration of displaced fluid elements).

We choose to study the dependence of the g-mode oscillation on the presence of density discontinuities for stellar models with a fixed mass $M = 1.4\ M_\odot$ because the mass of a neutron star is likely to be inferred from alternative observations (with respect to gravitational waves). The location of the high density discontinuity is chosen in such a way that the reasonable range of densities where first order phase transitions may arise is covered. In particular $10^{14}\ g/cm^3 \le \rho_\mathrm{d} \le 9\ 10^{14}\ g/cm^3$, while the amplitude $\Delta\rho/\rho_\mathrm{d}$ of the discontinuity is chosen in order to obtain the selected mass of the equilibrium model. When a density

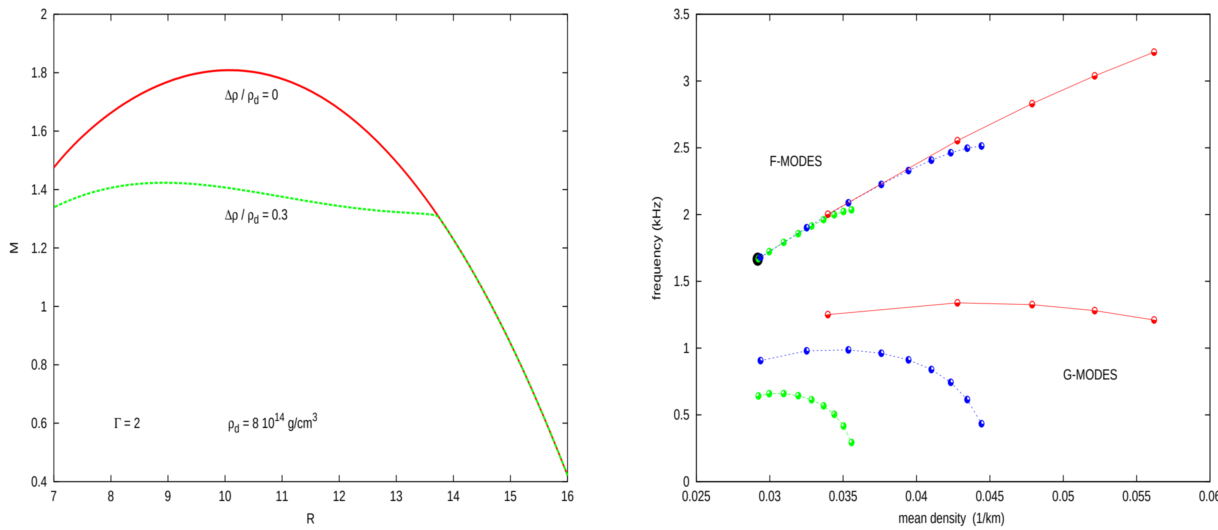

Figure 4: **Left panel:** The M-R relations for two models without density discontinuity (red) and with a discontinuity $\Delta\rho/\rho_\mathrm{d} = 0.3$ at $\rho_\mathrm{d} = 8\ 10^{14}\ g/cm^3$. **Right panel:** Frequencies of the f-modes and g-modes as a function of the average star's density. The fractional values of the discontinuities are of 10% (green), 20% (blue) and 30% (red), while the value of the density at the discontinuity cover the range $\rho_\mathrm{d} \simeq (1-9) \cdot 10^{14}\ g/cm^3$. All the models have $M = 1.4\ M_\odot$ and the same low-density equation of state. The black dot represents the f-mode frequency when no discontinuity is inserted in the equation of state.

discontinuity is introduced at high density in the equation of state, the global properties of the star are affected. The maximum allowed mass is lower for models with a discontinuity with respect to the case where no discontinuities are present. In addition, for a fixed mass, the star is more compact as it is shown in Figure 4 (left panel). In this case, the presence of the density discontinuity introduces a discontinuity g-modes, but also affects the usual picture of the pulsation spectrum. In particular the frequency of the fundamental mode of oscillation of a star depends on the mean density of the star itself [3, 4]. The f-mode frequency can be set into the approximate form $\omega_f^2 = k\, g \simeq l(l+1)\, M/R^3$. The variation of the star compactness due to the discontinuity shifts the f-mode frequency to higher values with respect to the "homogeneous" case. However, if the density jump is keep fixed, the radius of the $M = 1.4\ M_\odot$ neutron star increases with increasing values of the density at the discontinuity, resulting in a decreasing of the star mean density and in a subsequent decreasing of the f-mode frequency.

In Figure 4 (right panel) we show both the f-mode and the g-mode frequencies as a function of the average density of the star. The simultaneous detection of an f-mode and of a discontinuity g-mode could bring informations such as the amplitude of the discontinuity as well as its location. The value of the average density would be known from the f-mode detection and constraints could be made on the parameters of the discontinuity as well as on the radius of the star.

Tidal excitation of discontinuity g-modes of neutron stars in binary systems seems not to be a very efficient mechanism for the emission of gravitational radiation. This is due to the fact that g-modes have a small quadrupole moment (with respect to f-modes for instance) and the tidal interaction with an orbiting companion is then too small. We can not expect to detect g-mode resonances in the gravitational waves spectrum and waveforms with the first generation of detectors such as VIRGO and LIGO. A new class of very sensitive detectors is needed in order to detect them. We will focus on a new generation of gravitational detector, EURO, that has been recently proposed in a preliminary assessment study (http://www.astro.cf.ac.uk/geo/euro/). The assessment study envisages the noise, in the $10-10^4\ Hz$ range and for a shot-noise limited detector with a knee frequency $\nu_k = 100\ Hz$, to be

$$S_h(\nu) = 10^{-50}\,[\frac{3.6\ 10^9}{\nu^4} + \frac{1.3\ 10^5}{\nu^2} + 1.3\ 10{-}3\,(1+\frac{\nu^2}{\nu_k^2})]\ , \qquad (9)$$

while for a more ambitious Xylophone detector (which substantially reduces the shot noise)

$$S_h(\nu) = 10^{-50}\,[\frac{3.6\ 10^9}{\nu^4} + \frac{1.3\ 10^5}{\nu^2}]\ . \qquad (10)$$

In the following we will call these two possible configurations as EURO (9) and EURO-Xylo (10).

As we already noted, discontinuity g-modes due to first order phase transitions at high densities have frequencies that lie in the kHz region of the gravitational wave spectrum ($\nu_{\rm g} \approx (0.6-1.4)$ kHz). One of the mechanisms that may lead to the excitation of the high density discontinuity quasi normal modes is the onset of the phase transition during gravitational collapse or during the star's evolution. If sufficiently high density are reached in the core, a phase transition to quark matter could arise. The energy emitted in gravitational radiation is not expected to be more than $10^{-6}\ M\odot c^2$. In order to reach a significantly high signal to noise ratio, the collapse must be very close to us. If the source is at 10 Mpc, the signal to noise ratios for $\nu_{\rm g} = 1\ kHz$ would be

$$S/N(\mathrm{E}URO) \sim 1\ , \qquad S/N(\mathrm{E}URO - Xylo) \sim 80\ , \qquad (11)$$

while for galactic ($\sim 5\ kpc$) sources, that could be excited by pulsar glitches with an emitted energy of about $5\ 10^{-12}\ M_\odot$, the S/N ratio can be significantly larger, of the order of 4.5 and 360, for the two possible experimental configurations, respectively.

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8 Pedro Montero (SISSA)

Mini-CV

- 1999 MPhys Degree in Physics with Astrophysics
  University of Kent at Canterbury
  Canterbury, England
  Thesis: *Steady State Models of Interstellar Chemistry*

- 2000 MSc in Physics of the Atmospheric Environment
  University of Manchester Institute of Science and Technology
  Manchester, England
  Thesis: *Laboratory Studies of Ice Crystal Growth*

- 2001 2^{nd} Year PhD student in Astrophysics
  SISSA-International School for Advanced Studies
  Trieste, Italy

  PhD Project: Relativistic Dynamics of High Density Torus around a Black Hole

  A nuclear-matter density torus is expected to form around a rapidly rotating black hole after the coalescence of a binary system of neutron stars. This project will study the dynamics of this objects, the accretion onto the black hole and the emission of gravitational waves that will be produced.

Large Thick Disk Initial Data around a Black Hole
EU-Network Meeting (Southampton, 31/01-03/02/2002)

The analytical theory of accreting disk orbiting black holes (BHs) was first proposed by Kozlowski et al(1978).

Based on this theory and using the new developed EU-Hydro code (Whisky), this work aims to compute the initial data necessary to perform 3D simulations of the accretion of matter from a torus with pressure onto a rotating BH and the stability of this type of thick disc. In the first stage of this analysis the self-gravity of the torus will be neglected and the spacetime will be that of the black hole. Already with this simplified setup there are a number of interesting problems which we plan to investigate, such as non-axisymmetric oscillations of the torus and the runaway instability, which is an exponential mass loss that may occur when the accretion disc overflows the Lagrangian point.

We assume that the external gravitational field is stationary and axisymmetric. In this case, the metric does not depend on the time coordinate or on the azimuthal coordinate φ.

$$ds^2 = g_{tt}dt^2 + 2g_{t\varphi}dtd\varphi + g_{\varphi\varphi}d\varphi^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 \quad (12)$$

We also assume a perfect fluid stress-energy tensor, and a polytropic equation of state,

$$T_{\mu\nu} = (\epsilon + p)\, u_\mu u_\nu + p g_{\mu\nu} \tag{13}$$

$$p = k\rho^\gamma \tag{14}$$

Where p is the pressure, ϵ is the proper energy density and ρ, k, γ are the rest mass density, polytropic constant and the polytropic index. The specific angular momentum and the angular velocity are given by

$$l = -\frac{u_\varphi}{u_t} \tag{15}$$

$$\Omega = \frac{u^\varphi}{u^t} \tag{16}$$

From this equations it follows that,

$$l = -\frac{\Omega g_{\varphi\varphi} + g_{t\varphi}}{\Omega g_{t\varphi} + g_{tt}} \tag{17}$$

$$\Omega = -\frac{l g_{tt} + g_{t\varphi}}{l g_{t\varphi} + g_{\varphi\varphi}} \tag{18}$$

The equations of motion can be written as

$$\frac{\nabla_k p}{p + \epsilon} = -\nabla_k \ln u_t - \frac{\Omega \nabla_k l}{1 - \Omega l} \tag{19}$$

Note that the equations of motion simplifies considerably for the case of discs with constant angular momentum.

In order to construct a thick disc around a black hole we first calculate the metric coefficients in a Kerr background (only the Schwarzschild case is presented here); then we calculate the marginally stable and marginally bound orbits and the corresponding angular momentum of these orbits.

A disc with constant angular momentum, l_o, larger than the angular momentum of the marginally stable orbit and smaller than the one of the marginally bound orbit is chosen. This case corresponds to stable discs.

We then calculate the distribution $u_t(r, \theta)$ from the equations 4 and 5,

$$-u_t = \sqrt{\frac{g_{t\varphi}^2 - g_{tt} g_{\varphi\varphi}}{g_{tt} l^2 + 2 g_{t\varphi} l + g_{\varphi\varphi}}} \tag{20}$$

After calculating the inner and the outer radius of the disc and we initialize the hydrodynamical quantities.

Figure 1 shows the density on the equatorial plane for a disk with $l_o = 3.83M$ around a Schwarzschild black hole of one solar mass. Its inner and outer radius are located at $5.73M$ and $15.19M$ respectively. The cusp is located at $4.42M$. Note that if matter spills over the cusp then accretion is driven by pressure forces.

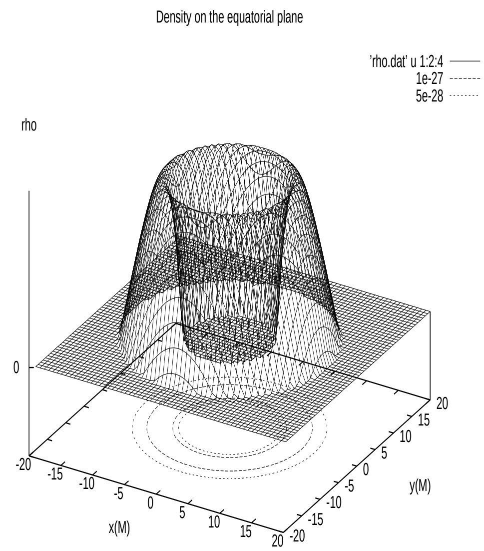

Figure 5: Density on the equatorial plane, case with $l_o = 3.83M$

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Relativistic irrotational fluids: 3D simulations in flat space

The following paragraphs will describe the project I am currently working on, whose final aim is to study the behaviour of relativistic irrotational fluids in a curved spacetime, such as Schwarzschild or Kerr. In order to do this, I am developing a code within the Cactus environment, to study numerically the problem. I present the recent results obtained in flat space, which show that the code is valid and ready to be extended to general metrics.

General equations

In order to define an irrotational fluid, we have to introduce the vorticity tensor:

$$\omega_{\mu\nu} = P^{\alpha}_{\mu} P^{\beta}_{\nu} \left[(hu_{\alpha})_{;\beta} - (hu_{\beta})_{;\alpha} \right]$$

where $P_{\alpha\beta}$ is the projection tensor, h is the enthalpy of the fluid and u^{μ} is the four velocity. The vorticity tensor is identically null for an irrotational fluid. From the Euler's equations for a perfect fluid:

$$(hu_{\mu})_{;\alpha} u^{\alpha} + h_{,\mu} = 0$$

we can derive a simple expression for the vorticity tensor which we write below :

$$\omega_{\mu\nu} = \left[(hu_{\mu})_{;\nu} - (hu_{\nu})_{;\mu} \right] \tag{21}$$

Equation 21 tells us that, if $\omega_{\mu\nu} \equiv 0$, then we can write the quantity hu_{μ} as the gradient of a scalar field, that is $hu_{\mu} = \phi_{,\mu}$. We then obtain the following expression for the conservation of particle density, in terms of ϕ :

$$\left[\left(\frac{n}{h} \right) \phi^{,\alpha} \right]_{;\alpha} = 0 \tag{22}$$

This equation must be matched with the one coming from the normalization condition for the four velocity u^μ:

$$h = \sqrt{-\phi^{,\alpha}\phi_{,\alpha}} \tag{23}$$

Equations 22 and 23 have been studied analytically in literature to study fluids in curved metrics such as Schwarzschild and Kerr. A first attempt to solve these equations can be found in [2] where equation 22 is linearized by setting $n = h$. This further approximation corresponds to the speed of sound being equal to the speed of light. A perturbative approach can instead be found in [1]. Our aim is to solve numerically equation 22, starting from the simple case of a flat space, and then moving to curved space-times, thus dropping the further approximation $c_s \equiv c$.

Thermodynamic considerations

We define the pressure to be given by the following polytropic expression:

$$p = kn^\gamma \tag{24}$$

Some straightforward thermodynamic calculations lead to the following equations which relate the particle density n to the enthalpy h.

$$\begin{aligned} h &= \frac{k\gamma}{\gamma - 1} n^{\gamma-1} + h_0 \\ n &= \left[(h - h_0)\frac{\gamma - 1}{\gamma k}\right]^{\frac{1}{\gamma-1}} \end{aligned} \tag{25}$$

The speed of sound can also be calculated and it turns out to be:

$$c_s = \sqrt{\frac{\partial p}{\partial \rho}} = \sqrt{\gamma - 1} \tag{26}$$

The conservation law system

In order to study numerically equation 22, we have to reduce it in first order conservation form (for further details, see [3] or [4]). We can do it for a flat space metric as follows: we write equation 22 in explicit form:

$$-\partial_t\left(\frac{n}{h}\partial_t\psi\right) + \partial_j\left(\frac{n}{h}\delta^{ij}\partial_i\psi\right) = 0 \tag{27}$$

We introduce the variables:

$$\begin{aligned} C &= \frac{n}{h}\partial_t\psi \\ D_i &= \partial_i\psi \end{aligned}$$

We thus obtain the following conservation law system:

$$\begin{aligned} \partial_t C - \partial_j\left[\frac{n}{h}\delta^{ij}D_i\right] &= 0 \\ \partial_t D_i - \partial_i\left[\frac{h}{n}C\right] &= 0 \end{aligned} \tag{28}$$

Equation 23 can be rewritten in term of the new variables in the following manner:

$$h^2 = \frac{C^2 h^2}{n^2} - \delta^{ij} D_i D_j \tag{29}$$

The conservation law system 28 is evolved numerically using a second order numerical method (Lax-Wendroff, Mac-Cormack). At the end of each timestep, the new value of h is computed using a Raphson-Newton iteration on equation 29. The value of n is then computed by using equation 25.

Numerical Results

In figures 6 and 7 some 1D results are shown. The evolution of an enthalpy perturbation with some initial velocity v_x has been studied. The code reproduces quite well our expectations for 1D evolution. We in fact expect the initial data to evolve following the characteristics given by the following expression:

$$\lambda_{12} = \frac{v_x \pm c_s}{1 \pm v_x c_s} \tag{30}$$

Following equation 30, we see that in figure 6, the initial data are subsonic, so that the initial perturbation is divided into two waves propagating in both directions. Figure 7 represents supersonic initial data, so the two characteristics are both in the same direction.

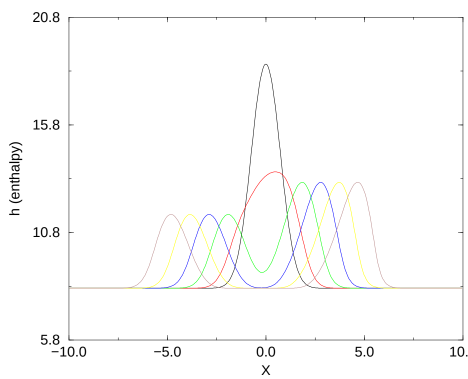

Figure 6: Fluid simulation with initial data: $c_s = 0.89$, $v_x = -0.24$

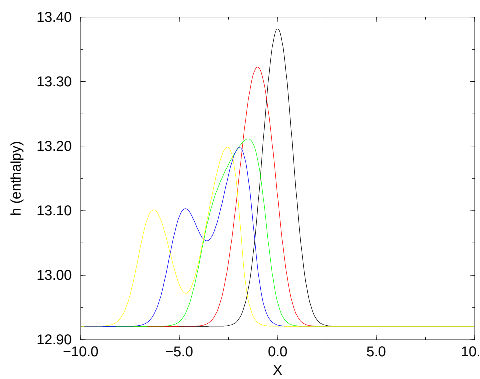

Figure 7: Fluid simulation with initial data: $c_s = 0.32$, $v_x = -0.60$

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10 José A. Pons (Rome)

Mini-CV

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- Postdoctoral Research Associate (1999-2000), State University of New York at Stony Brook.
- EU Network Postdoc (URLSDF) from 01/03/2001 to 30/11/2001
- Marie Curie fellow (URLSDF) since 01/12/2001
- The several general topics I have been working on are:
  1. General relativistic neutrino transport
  2. Equation of state of dense matter
  3. General Relativistic Hydrodynamics
  4. Neutron Star model atmospheres (Isolated NS RX J1856-3754)
  5. GR Stellar perturbations applied to coalescing binaries

GW from NS binaries: Effects of stellar structure

The group in Rome has been studying (in the perturbative regime, and the test particle limit) the effects that the structure of a neutron star would have on the gravitational emission of a binary system. By comparing the GW luminosity emitted by different stellar models with that of a black hole, we found that the stellar structure begins to affect the emitted power when the orbital velocity is $v > 0.2\ c$ ($\nu_{GW} > 185$ Hz for a naive extrapolation to a binary system composed of two $1.4 M_\odot$ neutron stars). These differences between different neutron star models and a black hole are found to be provoked by the excitation of the quasi-normal modes of the star.

We have chosen polytropic models for simplicity, but our results would be valid for general EOSs, because the differences in the gravitational flux are expected to depend more on global properties such as mass, radius, average density, or compactness (M/R), rather than on the specific matter distribution. The parameters we choose encompass a reasonable range of stellar models (radius ranging from 9 to 15 km), and the polytropic exponents, $\Gamma \equiv 1 + 1/n = 5/3$ and $\Gamma = 2$, cover most of the range of structural properties obtained with realistic EOS.

In Fig. 8 we plot the energy flux normalized to the Newtonian quadrupole, $P(v)$, as a function of the orbital velocity, for the models of star we have considered, and for a black hole. Sharp peaks appear if the central object is a star: they correspond to the excitation of the fundamental quasi-normal modes of the star for different values of the harmonic index l. A zoom of the region of interest (orbital radii larger than the ISCO and twice the stellar radius) is shown in Fig. 9

In Table 1 we show the values of the radius R_0, of the dimensionless orbital velocity v, and of the keplerian frequency ν_K of the orbit that corresponds to the excitation of the fundamental modes of the star for different l's for the considered stellar models. The corresponding frequencies of the f-mode are given in the last column. In Fig. 9, we show a zoom of Fig. 8 restricted to the region $v < 0.28$, which is far enough from the resonant

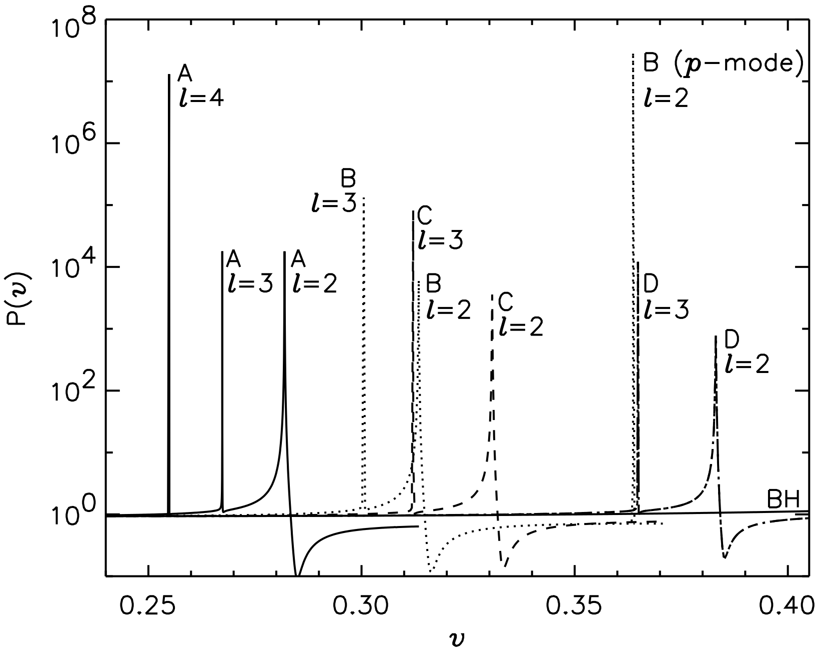

Figure 8: The normalized energy flux, $P(v)$, is plotted as a function of the orbital velocity for the stellar models and for a black hole. For model D and for the black hole the curves extend up to the velocity which correspond to the ISCO, whereas for the other models they stop when the mass m_0 reaches the surface of the star. The sharp peaks indicate that, for different values of the harmonic index l, the fundamental quasi-normal modes of the star are excited if the orbital frequency satisfies the resonant condition.

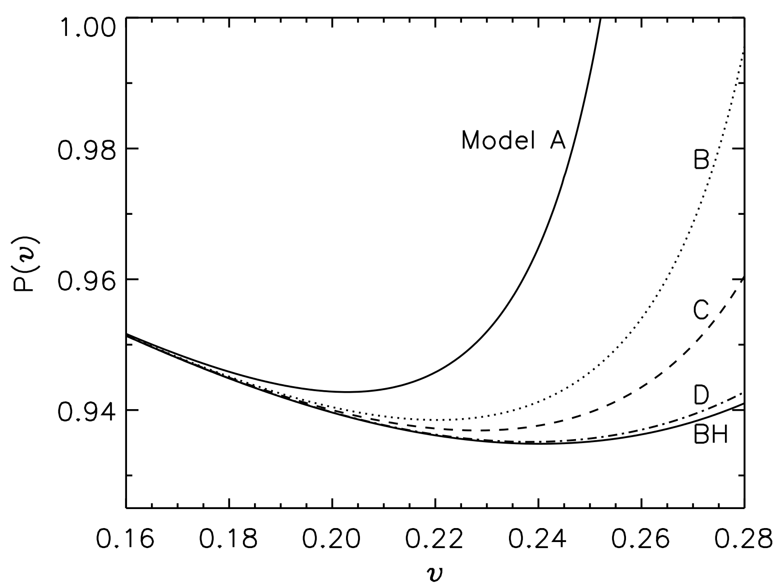

Figure 9: The normalized energy flux, $P(v)$, is plotted as in Fig. 8, but for a smaller orbital velocity range, such that the peaks due to the excitation of the stellar modes are excluded.

Table 1: In this table we give the values of the radius, orbital velocity and keplerian frequency (ν_K) of the circular orbits which correspond to the excitation of the fundamental mode of the considered stars for the first relevant multipoles, whose frequency is given in the last column. For model E we do not give these data for $l = 2$ because in order to excite the corresponding mode R_0 should be smaller than the ISCO ($6M$).

Model	l	R_0 (km)	v	ν_K (Hz)	ν_f (Hz)
A	4	31.8	0.255	567	2260
	3	29.0	0.267	651	1953
	2	26.0	0.282	767	1534
B	3	22.9	0.300	626	1879
	2	21.0	0.313	711	1422
C	3	21.2	0.312	702	2105
	2	18.9	0.331	835	1671
D	3	15.5	0.365	1119	3358
	2	14.1	0.383	1296	2593
E	3	13.5	0.391	1379	4138

orbits (except that for model A). In this case we can appreciate the differences between the emission of different stellar models and that of a black hole. Fig. 9 shows that the normalized energy fluxes emitted by different stellar models have a different slope, and are always larger than the flux emitted by the black hole. The curve of the more compact stellar model E, ($n = 1.5$, $R_0 = 9$ km), is practically indistinguishable from the black hole curve.

We are currently extending our work by computing the overlap function between Post-Newtonian (PN) templates and our gravitational signals in the test particle limit. The first results show that for very compact neutron stars, the use of PN approximants allows both the detection of the signal and an accurate (0.1 %) estimation of the mass of the system. However, for less compact stars, and for detectors very sensitive at high frequency around the kHz region (like the projected EURO), a large number of events may be missed if the structural effects are not properly taken into account. For the stiffer equations of state, the structural differences might be large enough to infer some properties of nuclear matter at supra-nuclear density from the gravitational signal during the late phases of the inspiralling.

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1997 – 2000:	PhD Thesis in Meudon, Paris. Advisor: Brandon Carter "Superfluid hydrodynamics in neutron stars"
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2000 - 2003:	(EU RTN) Postdoc at University of Southampton "Sources of Gravitational Radiation"
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Motivation

The study of neutron star oscillations is interesting as a possible source of gravitational waves for current and future gravitational wave detectors. However, when studying the *dynamics* of neutron stars, one faces many complications due to the complexity of these objects, which makes it necessary to use more sophisticated models for their description than simple perfect fluid stars. One would have to allow for a solid (but elastic) crust of about 1 km thickness, very high magnetic fields (up to 10^{14} gauss), superfluidity of neutrons and protons and the ensuing complexities of vortex dynamics and their interactions, hyperons and other exotic particles in the inner core, just to name some of the additional ingredients necessary to get a physically satisfying picture. Eventually one also has to use a fully generally relativistic model for quantitatively correct predictions.

Our general strategy is to add these additional pieces of physics one at a time in order to get a good understanding of their effects and importance, and we started by including a feature that is expected to be one of the most relevant corrections to the description of the neutron star's behaviour, namely the superfluidity of the neutrons, which form the bulk of the neutron star matter.

Superfluidity in neutron stars is a very "stable" prediction, which even predates (Migdal 1959 [1]) the actual discovery of neutron stars in 1968. This prediction is based on the enormous degeneracy of the neutrons due to the density of neutron stars (nuclear density and above, $\rho > 10^{14}\mathrm{g/cm}^3$), and has been confirmed up to today by the most recent calculations of nuclear physics (see eg. [2]).

The second indication of the actual importance of superfluidity in the dynamics of neutron stars comes for the observation of the "glitch" phenomenon in pulsars: a sudden rapid spin-up of up to $\Delta\Omega/\Omega \sim 10^{-6}$ on timescales of minutes, followed by a very slow (on timescales of months) relaxation towards the initial equilibrium slowdown. All current glitch models are based on the idea that a superfluid inside the neutron star can lag behind the rotation rate Ω of the crust (and the magnetic field anchored to it). Once this lag becomes greater than a certain threshold, an instability develops and leads to a "catastrophic" transfer of angular momentum from the faster spinning neutron superfluid onto the crust, resulting in the observed spin-up. The following relaxation on extremely long timescales also indicates the very weak and peculiar type of coupling (mediated by neutron vortices) between these two components.

Recent and current projects

The first project concerned the Newtonian equilibrium structure of a stationary superfluid neutron star. The model used is the standard "two–fluid model", in which we allow the superfluid neutrons to spin at a different rate Ω_n from the "normal" comoving components (sometimes called "protons" for simplicity) rotating at Ω_c. This study is interesting by itself, on one hand because it represent the initial state of practically all current glitch–models, but also because it can serve as the unperturbed background in an oscillation mode study of rotating neutron stars. This project [3] was a collaboration with G. Comer of St. Louis University and N. Andersson, and besides finding an analytic solution for the background in the slow–rotation approximation (suitably generalised for two interacting fluids), we where also able to show the great importance of the two types of interaction between the two fluids: "symmetry energy" (characterised by σ) and "entrainment" (parameter ε). A clear demonstration of this is seen in Fig. 10, showing a non–interacting configuration with the usual oblate structure, while for certain interaction parameters we can find the protons in a "prolate" configuration despite their rotation.

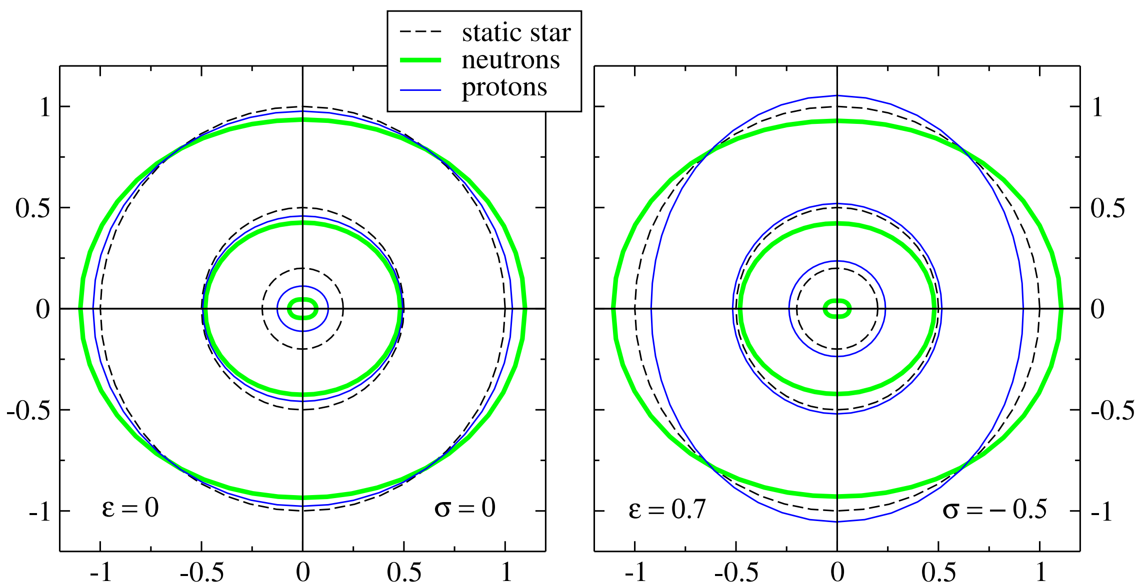

Figure 10: Iso–density profiles of neutrons and "protons" spinning at different rotation rates $\Omega_n \neq \Omega_c$ for non–interacting fluids (left panel) and with certain (physically allowed!) interaction parameters σ and ε.

Currently I am working [4] with J. Novak (Meudon) and G. Comer at a fully relativistic generalisation of this analysis, using the LORENE library for numerical relativity developed by the Meudon group. The code is now well advanced and in a testing state.

A project started at my recent 2 months visit in Toulouse is now close to finished, in which I studied together with M. Rieutord the oscillation spectrum of a simple non–rotating superfluid neutron star [5]. We were able to clearly confirm the presence of a new family of modes (dubbed "superfluid modes"), in which the two fluids are moving in opposite phase (see Fig. 11), and again, the interaction parameters between the two fluids are essential for the frequencies and structure of the modes, which can also result in the well–known "avoided crossing" phenomenon between "ordinary" and "superfluid" modes. Close to an "avoided crossing", the clear-cut distinction between "ordinary" and "superfluid" mode seen in Fig. 11 becomes "blurred" and after the crossing, the two modes have changed character. Another interesting phenomenon is the disappearance of the g–modes in the superfluid case, which would be present for the same background configuration if the two components where "normal" and forced to move together.

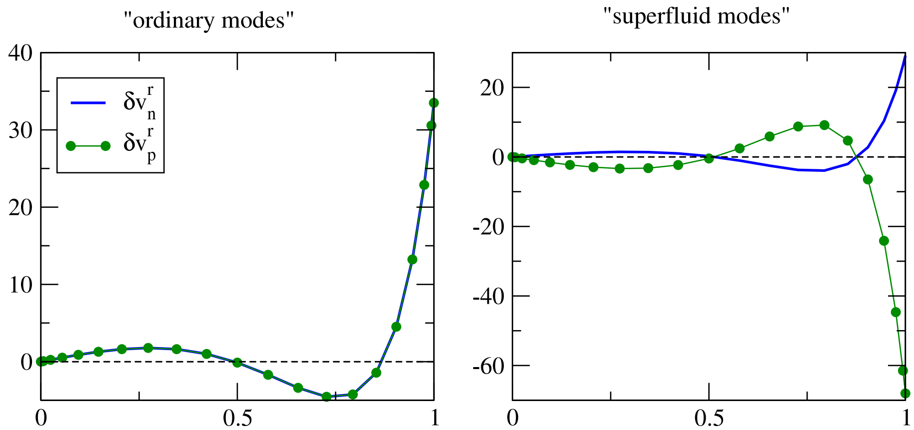

Figure 11: An "ordinary" and "superfluid" p_2 mode, in which the two fluids are either moving in phase or in opposite phase.

Finally, in collaboration with N.Andersson and G.Comer, we are currently working on moving this analysis to the more interesting case of rotating neutron stars. This case is particularly interesting because of the presence of inertial modes (which are absent in the non–rotating case), which are the most interesting candidates so far for the emission of gravitational waves of oscillating neutron stars.

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How to invariantly characterize non linear black hole perturbations

The aim of this work is to introduce a characterization of a non linearly perturbed black-hole spacetime in vacuum, using an approach based on the Weyl curvature scalars. The physical background is the Bondi Sachs metric:

$$\begin{aligned} ds^2 &= -\left(\left(1-2\frac{M}{r}\right)e^{2\beta}-U^2r^2e^{2\gamma}\right)dv^2+2e^{2\beta}dvdr \\ &+ 2Ur^2e^{2\gamma}dvd\theta+r^2(e^{2\gamma}d\theta^2-e^{-2\gamma}\sin\theta^2d\phi^2) \end{aligned} \tag{31}$$

where the metric coefficients (β, M, γ, U) depend on the variables (u, r, θ). This metric describes an axisymmetric non rotating spacetime and, from a purely physical viewpoint, it can be considered as a "perturbation" of a spherical black-hole described by the Schwarzshild metric. In [1] the metric (1) is used to compute the black-hole response to the external perturbation. This response is encoded in a superposition of secondary generated harmonics and the metric unknown coefficients, as well as the amount of energy emitted further to the perturbation, are numerically calculated at each point of the spacetime in the Bondi code [2]. Our purpose is to characterize this non linear perturbations using the Weyl scalars. These are defined as the projection of the Weyl tensor on an arbitrary Newmann-Penrose null tetrad. With respect to these tetrad, it is possible to give an algebraic description of the gravitational field, the *Petrov classification*, that summarizes the physical characteristics of the different spacetimes in Petrov types. A physical interpretation of the latter is given in [3]. According to this, for the Bondi-Sachs metric that is Petrov Type I, no particular physical characteristic emerges, unless it is possible to fall in one of the so-called Standard forms for Petrov type I. From this point of view, the physically interesting case is the one in which the scalars Ψ_1, Ψ_3 are equal to zero, because they represent only gauge field. In this situation the gravitational field is composed by a Coulomb field (Ψ_2) with an ingoing (Ψ_0) and an outgoing (Ψ_4) transverse non linear wave superimposed. Ψ_2 would be the only non-zero scalar for the unperturbed black-hole.

In order to obtain this representation, the work plan I am currently developing is to calculate, for the given metric, a Newmann-Penrose null tetrad. Knowing this, it is possible to calculate the five Weyl scalars, that are all non zero, given the complete arbitrary of the tetrad frame. In order to obtain the standard form of interest for the Bondi-Sachs metric, we need to use the three classes of rotations for the tetrad vectors [4]. Once obtained the rotated scalars Ψ_0, Ψ_2, Ψ_4, the aim is to explicitly calculate them at each point of the spacetime using the Bondi code in order to obtain an invariant characterization of the curvature throughout the spacetime.

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1998 - 2001	PhD, Faculty of Mathematical Studies, University of Southampton, "Non-linear numerical schemes in General Relativity"
2001 -	Post-doctoral position in Thessaloniki, Greece, EU-network project "Sources of Gravitational waves"

Non-linear perturbations of neutron stars: Radial oscillations in a Lagrangian formulation

We present a new numerical scheme for the evolution of non-linear evolutions of neutron star oscillations in the case of radial oscillations in a Lagrangian formulation. The key feature of the scheme is to view the evolution of the physical variables as deviations from an equilibrium background configuration and thus eliminate numerical contaminations arising from the presence of background "zero-order" terms in the evolution equations.

A non-linear perturbative numerical scheme

We start by illustrating the basic principle of our new numerical approach in the case of a schematic differential equation. Let us assume that a physical system is described in terms of variables $\hat{f}$ and $\hat{g}$, which are functions of time t and spatial position r, and that one of the partial differential equations (PDE) determining the evolution of the system is given by

$$\hat{f}_{,t} = \hat{g}_{,r} - \hat{f}\hat{g}. \tag{32}$$

The only prerequisite for applying our method is the existence of a non-trivial equilibrium configuration of the system in which Eq. (32) reduces to

$$0 = g_{,r} - fg. \tag{33}$$

Here we have omitted the "hat" in denoting the equilibrium functions in order to distinguish them from their dynamic counterparts. Next we decompose the time dependent functions into equilibrium "background" contributions and time dependent deviations according to

$$\hat{f}(t,r) = f(r) + \delta f(t,r), \tag{34}$$

$$\hat{g}(t,r) = g(r) + \delta g(t,r). \tag{35}$$

We emphasize that in contrast to conventional perturbation theory no assumptions are made with respect to the size of the deviations. If we insert Eqs. (34), (35) into the original PDE (32), we obtain

$$\delta f_{,t} = g_{,r} - fg + \delta g_{,r} - f\delta g - g\delta f - \delta f\delta g. \tag{36}$$

The crucial terms in this equation are the first two on the right hand side: $g_{,r} - fg$. From Eq. (33) we know that these terms cancel each other identically. Numerically, however, this will in general only be satisfied up to a residual numerical error which constitutes a spurious source term in the numerical evolution of $\hat{f}$ or δf according to Eq. (32) or (36). This numerical contamination is particularly severe for deviations significantly smaller than the background values, but sufficiently large to warrant non-linear effects. Aside from such accuracy issues it has been demonstrated in [1] that the residual numerical error may lead to spurious physical phenomena such as the collapse or evaporation of a stable neutron star. We therefore make use of our knowledge about the equilibrium configuration and eliminate the first two terms on the right hand side of Eq. (36) *before* the numerical implementation. We thus obtain the final evolution equation

$$\delta f_{,t} = \delta g_{,r} - f\delta g - g\delta f - \delta f \delta g, \tag{37}$$

which provides a fully non-linear description of the problem equivalent to Eq. (32).

Dynamic spherically symmetric neutron stars in Lagrangian gauge

We will now apply the numerical scheme of the previous section to the evolution of non-linear radial oscillations of neutron stars. In order to facilitate an exact description of the surface of the star, we use a Lagrangian gauge in which the radial coordinate is comoving with the fluid elements. Following [2] we further impose the polar slicing condition and thus arrive at the line element

$$ds^2 = -\hat{\lambda}^2 \left(1 - \frac{w}{\hat{\Gamma}}\right) dt^2 + 2\frac{\hat{r}_{,x}\hat{\lambda}w}{\hat{\Gamma}} dt\, dx + \frac{\hat{r}_{,x}^2}{\hat{\Gamma}} dx^2 + \hat{r}^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{38}$$

where $\hat{\lambda}$, $\hat{\Gamma}$, $\hat{r}$ and $w = \hat{\lambda}\hat{r}_{,t}$ are functions of time t and radial position x. We model the neutron star as a perfect fluid at zero temperature with a polytropic equation of state $\hat{P} = \kappa\hat{\rho}^{\gamma}$. The resulting field and matter equations as well as the corresponding formulation in terms of deviations analogous to Eq. (37) can be found in [1] and is solved with a second order in space and time implicit scheme similar to the Crank-Nicholson method. In the static limit these equations reduce to the Tolman-Oppenheimer-Volkoff equations the solution of which serves as our non-trivial equilibrium background.

We have tested the performance of the resulting code in three independent ways. First we have simulated the Oppenheimer-Snyder dust collapse of a pressureless, initially homogeneous fluid. Next we have tested the code for low amplitude oscillations where the exact solution is approximated with high accuracy by the solution of the linearized problem. In both cases we obtain a relative accuracy of 10^{-4} using 800 grid points. Finally a time dependent convergence analysis demonstrates second order convergence of the code for stellar oscillations with larger amplitudes corresponding to $\Delta R/R \approx 0.01$. The details and graphical representations of these tests can be found in [1].

Non-linear mode coupling

We have used the amplitude independent accuracy provided by our new scheme to investigate the coupling of eigenmodes due to non-linear effects. For this purpose we evolve an isolated eigenmode j with amplitude K_j measured in terms of the surface displacement in metres. Following [3] we monitor the presence of other eigenmodes during the non-linear evolution by utilizing the fact that the linearized equations can be formulated in the form of a self-adjoint eigenvalue problem in terms of the rescaled displacement variable $\zeta = r^2\Delta r/\lambda$ [see for example Misner, Thorne & Wheeler (1973)]. By virtue of the completeness and orthogonality

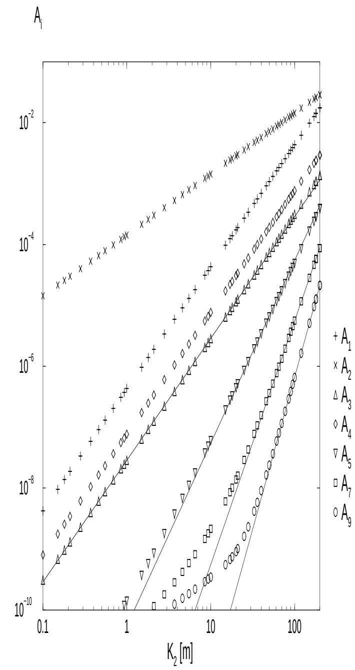

Figure 12: The eigenmode coefficients A_1-A_5, A_7 and A_9 as functions of initial amplitude K_2. The values for A_6 and A_8 are close to those of A_5 and A_7 respectively and have been omitted for clarity. The solid lines represent power laws $\sim K_2^n$ for $n = 2, 3, 4, 5$ from left to right. We attribute deviations from these power laws at small values of A_i to numerical noise.

properties of the resulting set of eigenfunctions ζ_i we can expand the displacement $\zeta(t, r)$ obtained from the non-linear evolution in a series of the eigenfunctions according to

$$\zeta(t, r) = \sum_{i=1}^{\infty} A_i(t)\zeta_i(r), \tag{39}$$

where the eigenmode coefficients are given by the inner product $A_i(t) = \langle\zeta(t, r), \zeta_i(r)\rangle$ as defined by the eigenvalue problem (see [1] for details). The maximum of $A_i(t)$ obtained for an evolution over about $10^3\,M$ corresponding to about 10 oscillations of the fundamental mode is then taken as a measure of the degree of excitation of mode i. In Fig. 12 we show results thus obtained for a neutron star model with radius $R = 11.3\,\text{km}$, mass $m = 1.48\,M_\odot$ and a polytropic equation of state with $\gamma = 2$ and $\kappa = 150\,\text{km}^2$ and initial data in the form of the second eigenmode profile for the fluid displacement. The results demonstrate the increasing excitation of higher order modes with increasing amplitude. We further observe that the dependence of the eigenmode coefficients A_i on the initial amplitude can be modeled by power laws of increasing integer index as indicated in the figure. In future work these features as well as the dependence of the mode coupling on stellar parameters will be investigated in more detail. The Lagrangian code based on our new numerical scheme will also serve as the background for studying perturbations of slowly rotating collapsing stars.

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Another twist in the f-mode instability — differential rotation of stars

The occurrence of an f-mode instability in a newly born uniformly rotating neutron star is in general prevented by the large bulk and shear viscosities of nuclear matter in those conditions and detailed calculations have shown that the instability is suppressed except for very large rotation rates, close to the mass-shedding limit [9]. However, there are elements which have improved our understanding of the instability and have again increased the expectations that the f-mode instability might characterize the earliest life stages of a newly formed neutron star. The first of these new elements was provided by [4, 18, 16] who have shown that, general relativistic effects tend to further destabilize the f-mode, lowering the critical value of the ratio between the stellar rotational kinetic energy and the absolute value of the gravitational energy, $\beta_c \equiv (T/|W|)_c$ at which the secular f-mode instability is triggered. The second element was provided by [15], whose fully general relativistic hydrodynamical simulations have shown that the remnants of binary neutron star mergers could be, at least for polytropic equations of state, rapidly and differentially rotating stars. In addition to this, [12] have recently computed the structure of objects formed in accretion-induced collapse of rotating white dwarfs and found that these objects can rotate extremely rapidly and differentially.

In this revised picture, we have computed the eigenfrequencies of f-modes and determined the secular stability limits for rapidly rotating relativistic stars with differential rotation.

Stellar model

The equilibrium stellar models are assumed to be stationary and axisymmetric and are constructed with a numerical code based on the method of [10]. In order to investigate how the limits for the secular instability depend on the "stiffness" of the EOS, we have performed calculations for two different polytropic models whose properties are summarized in Table I. Along each sequence of rapidly rotating stellar models, the polytropic constant κ, the polytropic index N and the total rest-mass M_0 are kept constant.

	N	κ	$M_0/M_\odot$
Model (a)	0.5	6.02×10^4	1.60
Model (b)	1.0	1.00×10^2	1.52

Table 2: Properties of the polytropic equilibrium models. Equation of state is that of polytrope, $p = \kappa\rho^{1+1/N}$. We adopt units in which $c = G = M_\odot = 1$.

The rapidly rotating stellar models are constructed after specifying a law of differential rotation $\Omega = \Omega(r, \theta)$ with a choice which is, to some extent, arbitrary. This is because there are a number of differential rotation laws that satisfy the integrability condition of the equation of hydrostatic equilibrium as well as the Rayleigh criterion for dynamical stability [17]. All of these laws are physically consistent, cannot be excluded on physical grounds and might influence the qualitative behaviour of the results. We here follow the formulation suggested by [10] who have modeled the rotational angular velocity profile as

$$A_{\rm R}^2(\Omega_0 - \Omega) = \frac{(\Omega - \omega) r^2 \sin^2\theta e^{2(\mu-\nu)}}{1 - (\Omega - \omega) r^2 \sin^2\theta e^{2(\mu-\nu)}} \ . \tag{40}$$

Here Ω_0 is the angular velocity at the rotational axis and $A_{\rm R}$ is a dimensionless parameter accounting for the degree of differential rotation. In particular, the degree of differential rotation increases with

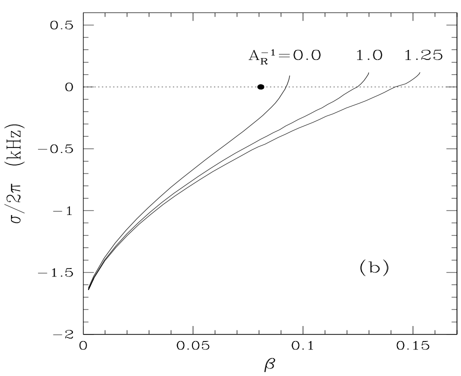
A_R^{-1}, and in the limit of $A_R^{-1} \to 0$, the profile reduces to that of uniform rotation. In the Newtonian limit, the differential rotation law (40) reduces to the so-called "j-constant" law [3, 6].

The oscillation modes on the numerically-obtained equilibrium models are investigate by the same procedure as in [18]. Note that we are working within relativistic Cowling approximation [13].

For each degree of differential rotation A_R and for each mode-number m, the solution to the eigenvalue problem is found for increasing values of β until the limit of mass-shedding is found, which is indicated with β_s (see [10] for a definition of β).

Results

The results of these calculations are presented in Fig.1, where we plot the frequencies of the $m = 2$ mode as a function of the parameter β for the model (b) of Table I. Different curves refer to different degrees of differential rotation. Note that the values of β at which $\sigma = 0$ (i.e. β_c) signal the onset of the secular instability (neutral points) and that these increase as the degree of differential rotation is increased. At the same time, however, the differentially rotating models are able to support larger values of β before reaching the mass shedding limit β_s, represented by the end-points of the curves in Fig.1.



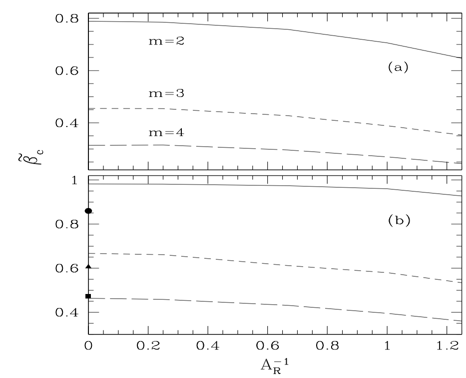


Figure 13: [Left] Eigenfrequencies of the $m = 2$ mode as a function of the parameter $\beta = T/|W|$ for the model (b) of Table I. Different curves refer to different degrees of differential rotation, with the $A_R^{-1} = 0.0$ line being the one of a uniformly rotating model. The filled dot indicates the neutral stability point of a uniformly rotating star computed in full general relativity [16].

Figure 14: [Right] $\widetilde{\beta}_c$ as a function of the rate of differential rotation. The two panels refer to the two models of Table I and the different line types refer to different mode-numbers. The filled symbols show a comparison with the fully general relativistic calculations of [16].

Given the results of Fig.1 it is natural to ask whether differential rotation favours or not the onset of the f-mode instability in rapidly rotating relativistic stars. The evidence that β_c increases with increasing differential rotation is not sufficient to draw a conclusion since differentially rotating models can support values of β which are considerably larger than those of uniformly rotating stars before reaching the mass-shedding limit. This is simply due to the fact that in the differentially rotating model described by eq. (40) the inner regions can rotate rapidly while the outer regions rotate more slowly, preventing mass-shedding. Thus, to quantify the importance of differential rotation for the onset of the instability we need to measure not only the secular stability limit β_c, but also how close the latter is to the ultimate limit of mass-shedding β_s. A relative measure of the two quantities across sequences with different degrees of differential rotation can provide information on the the existence of a configuration at the onset of the secular instability and clarify the role played by differential rotation. In this sense, the normalization of β_c with β_s is equivalent to the normalization, commonly used for uniformly rotating stars, of writing the stellar angular velocity in terms of the mass-shedding angular velocity.

In Fig.2 we show the behaviour of the ratio $\widetilde{\beta}_c \equiv \beta_c/\beta_s$ as obtained for degrees of differential rotation ranging between $A_{\rm R}^{-1} = 0$ (uniform rotation) to $A_{\rm R}^{-1} = 1.25$.

The two panels of Fig.2 refer to the models (a) and (b) of Table I, respectively. In each panel, the different lines refer to the mode-numbers $m = 2, 3$ and 4. As anticipated above, the Cowling approximation overestimates the stability of the stellar models by introducing an error which is of the order of 10% for the $m = 2$ mode but is considerably smaller for higher mode-numbers. This can be appreciated by comparing with the fully general relativistic values of $\widetilde{\beta}_c$ computed for uniformly rapidly rotating stellar models [16] and indicated with filled symbols in the lower panel of Fig.2. The overall decrease of $\widetilde{\beta}_c$ for increasing degrees of differential rotation provides a clear indication that the onset of the secular f-mode instability is in general *favoured* by the presence of differential rotation. This is most evident in the case of a stiffer EOS [model (a) of Table I] where, for reasonable degrees of differential rotation, $\widetilde{\beta}_c$ is reduced by about 17%.

A more detailed discussion of the results presented in this paper can be found in [19].

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