Reinhard PRIX University of Southampton

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On the two–fluid neutron star model

Plan

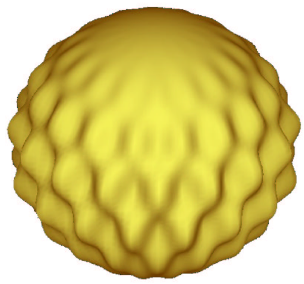
I. Motivation of the two–fluid model

II. Two–fluid hydrodynamics

III. The Stationary Two–fluid NS model

IV. Outlook

I.Motivation



Neutron star oscillation modes

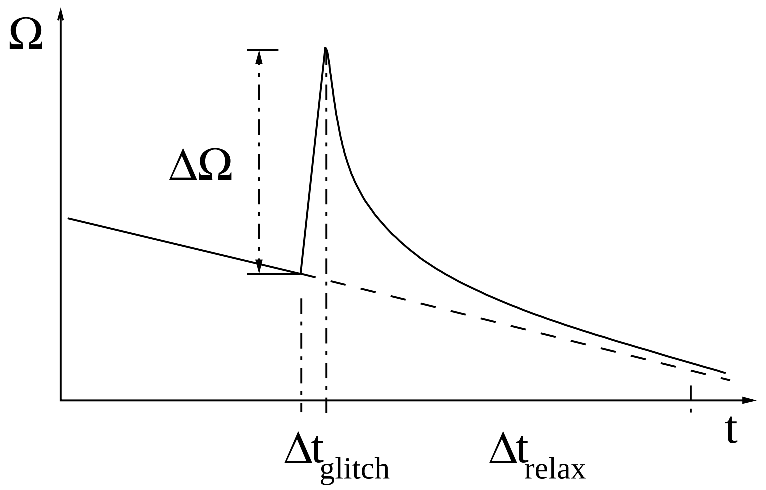
$\longrightarrow$ Gravitational wave (GW) emission

$\longrightarrow$ GW driven instabilities: eg. r–modes

N.Anderssson; J.Friedman, S.Morsink (1998); ...

! most oscillation mode studies: single perfect fluid (Newtonian)

BUT: real pulsars show various rotation irregularities, eg. "Glitches":



$\Delta\Omega/\Omega \;\sim\; 10^{-6} - 10^{-9}$

$\Delta t_{\mathrm{glitch}} \sim$ minutes

$\Delta t_{\mathrm{relax}} \sim$ months

internal "reservoir" of angular momentum: very weakly coupled

$\longrightarrow$ superfluid? $\Omega_S > \Omega$ as $\Omega \downarrow$

instability: $\Omega_S \to \Omega \Longrightarrow$ Glitch

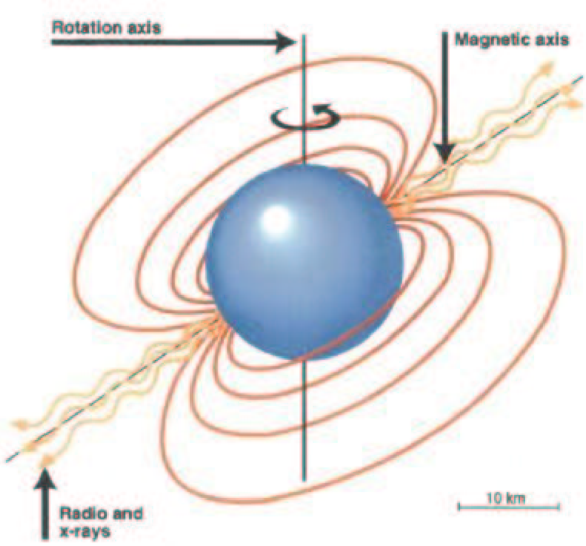
Observational motivation for

☞ SUPERFLUID

☞ Two–fluid NS model: $\Omega_S \neq \Omega$

“Astro–seismology” of neutron stars?

! detailed understanding and modeling of the internal NS dynamics is necessary, even prior to detection of oscillations!
(contrary to helio–seismology)



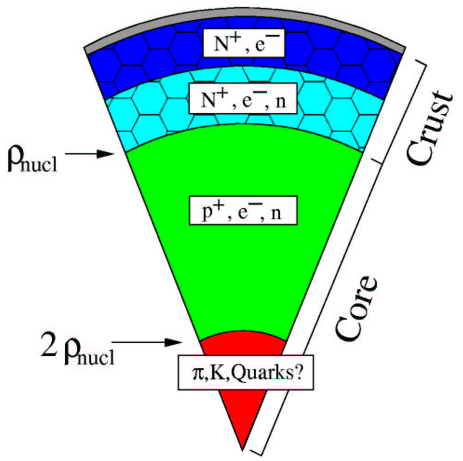
some necessary ingredients:

- General Relativity ($\sim 40\%$ corrections)
- magnetic field ($B \sim 10^{12}$ G)
- solid crust (viscous boundary layer?)
- superfluidity + superconductivity

☞ vortices

☞ two (or more) fluids

Nuclear Physics: Internal structure



extremely degenerate matter:

$$T \lesssim 10^8 \text{ K} \quad \ll \quad T_{\rm F} \sim 10^{12} \text{ K}$$

! thermal effects negligible

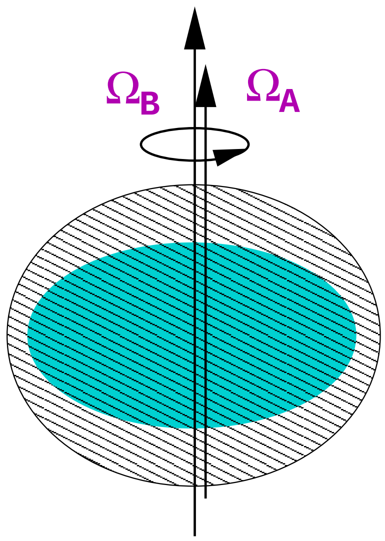
☞ superfluid neutrons

☞ superconducting protons

(transition temperature $T_{\rm c} \sim 10^{10}$ K)

Simplest non–trivial model: Two–fluid NS

- Fluid A: Superfluid neutrons (no viscosity!)
- Fluid B: all charged components (protons, electrons, muons, ...), coupled by magnetic field and viscosity



❑ can have $\Omega_{\rm A} \neq \Omega_{\rm B}$

❑ base of all current glitch–models

❑ fluids A and B strongly coupled!

☞ "entrainment"

II. Two–fluid hydrodynamics

B.Carter: covariant canonical hydrodynamics

particle current: $\vec{n}$ particle density: $n = \sqrt{-\vec{n}^2}$

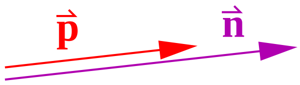
particle conservation: $\nabla_\alpha n^\alpha = 0$

energy density: E momentum/particle: $p_\alpha \equiv -\partial E / \partial n^\alpha$

equation of motion: $$n^\alpha \, \nabla_{[\alpha} p_{\beta]} = 0$$

❏ one fluid case: barotropic equation of state $E = E(n)$

$\Longrightarrow \ \vec{p} \parallel \vec{n}$



...leads to usual hydro

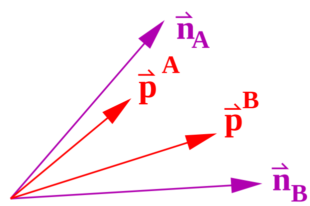
❏ two fluids A, B:

conserved currents: $\vec{n}_{\mathrm{A}}, \ \vec{n}_{\mathrm{B}},$ relative velocity: Δ

particle momenta: $\vec{p}^{\mathrm{A}}, \ \vec{p}^{\mathrm{B}},$ $p^{\mathrm{A}}_\alpha \equiv -\partial E / \partial n^\alpha_{\mathrm{A}}\,, \quad p^{\mathrm{B}}_\alpha \equiv -\partial E / \partial n^\alpha_{\mathrm{B}}\,,$

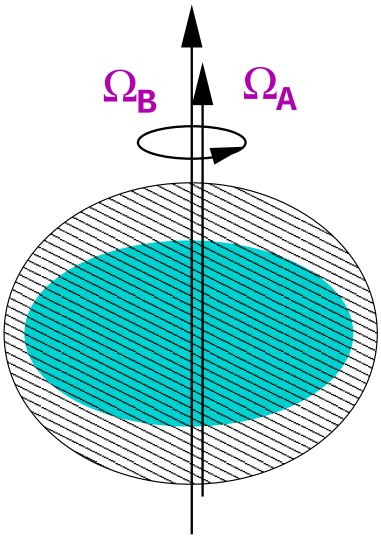
Equation of state: $E = E\left(n_{\mathrm{A}}, \, n_{\mathrm{B}}, \, \Delta^2\right)$

$$\Longrightarrow \quad \begin{aligned} \vec{p}^{\mathrm{A}} &= K^{\mathrm{AA}} \, \vec{n}_{\mathrm{A}} + K^{\mathrm{AB}} \, \vec{n}_{\mathrm{B}} \\ \vec{p}^{\mathrm{B}} &= K^{\mathrm{BA}} \, \vec{n}_{\mathrm{A}} + K^{\mathrm{BB}} \, \vec{n}_{\mathrm{B}} \end{aligned}$$



! "ENTRAINMENT" ... consequence of the interaction A $\leftrightarrow$ B

III. Stationary Two–fluid NS model



uniform rotations $\Omega_A \neq \Omega_B$

Coupling A $\leftrightarrow$ B by gravitational field and by entrainment!

previous projects: slow–rotation expansion

R.Prix, A&A **352**,623 (1999) (Newtonian)

N.Andersson, G.Comer, CQG **18**, 969 (2001) (GR)

$\rightarrow$ entrainment not fully included!

(A) Newtonian slow rotation expansion WITH entrainment

R.Prix, G.Comer, N.Andersson, in preparation

Two equations of motion + stationarity + axisymmetry:

$\Longrightarrow$ Two integrals of motion:

$$\mu_A + m_A\Phi - \frac{1}{2}m_A{v_A}^2 = \text{const}_A$$
$$\mu_B + m_B\Phi - \frac{1}{2}m_B{v_B}^2 = \text{const}_B$$

+ Poisson equation: $\nabla^2\Phi = 4\pi G\rho$

Slow Rotation: Taylor expansion in Ω_A and Ω_B up to $\mathcal{O}(\Omega^2)$:

$$\text{eg. } \Phi(r,\theta) = \Phi^{(0)}(r) + \underbrace{\delta\Phi(r,\theta)}_{\mathcal{O}(\Omega^2)} + \mathcal{O}(\Omega^4)$$

☞ numerical solution (ODE) for any given EOS: $E = E(n_A, n_B, \Delta^2)$

! analytic solution $\mathcal{O}(\Omega^2)$ for a particular class of EOS:

$$E = \underbrace{(\kappa_A\, n_A{}^2 + \kappa_B\, n_B{}^2)}_{\text{sum of two } \gamma = 2 \text{ polytropes!}} + 2\sigma\, n_A n_B + \underbrace{\beta n\, \Delta^2}_{\text{entrainment}}$$

How good is this entrainment model?

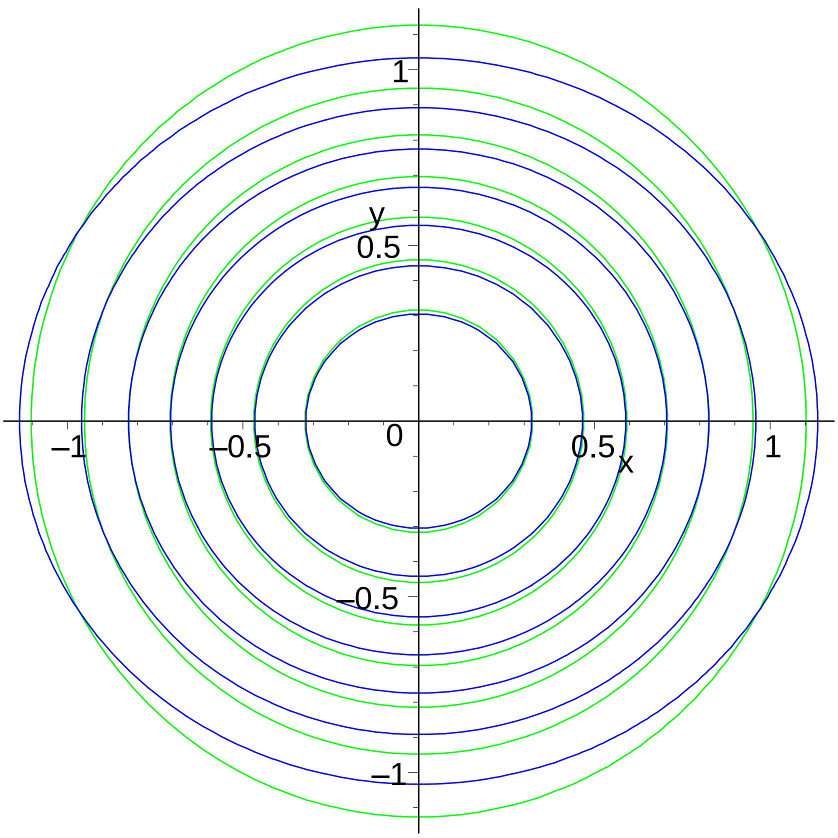
equivalence "entrainment" $\longleftrightarrow$ "effective masses" m^*:

$$! \ \beta = \text{const.} \quad \Longleftrightarrow \quad m_p^* = \text{const.}!$$

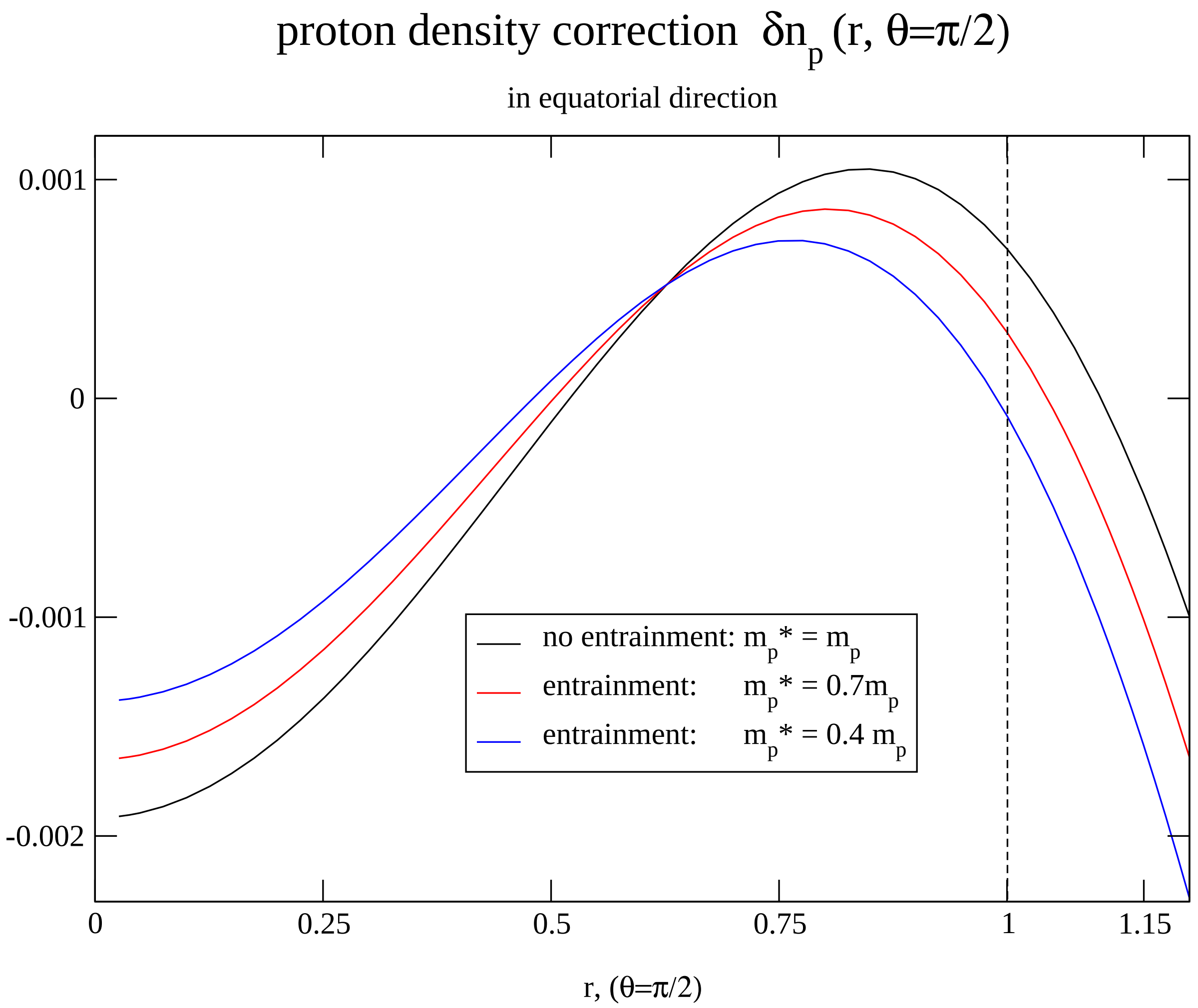
nuclear physics: $m_p^* \approx 0.6\, m_p$ and slowly varying with density!

Some plots of the analytic solution:

isodensity surfaces of n_A and n_B ($\Omega_A \approx 4000\ \text{s}^{-1}$, $\Omega_B \approx 0.5\, \Omega_A$):



$$n_{\mathrm{p}}(r,\,\theta) = n_{\mathrm{p}}^{(0)}(r) + \delta n_{\mathrm{p}}(r,\,\theta) + \mathcal{O}(\Omega^4)$$



(B) "Exact" numerical solution in GR

current project with J.Novak (Meudon) and G.Comer

2 Killing vectors: stationarity ... $\vec{t}$ axisymmetry ... $\vec{\varphi}$

most general metric in MSQI gauge: (N ... lapse, N^φ ... shift)

$$ds^2 = -(N^2 - N_\varphi N^\varphi)\,dt^2 - 2N_\varphi\,d\varphi\,dt + A^2\left(dr^2 + r^2 d\theta^2\right) + B^2 r^2 \sin^2\theta\,d\varphi^2$$

symmetries $\vec{t}$, $\vec{\varphi}$ $\Longrightarrow$ two first integrals:

$$\log N + \log \mu_{\rm A} - \log \Gamma_{\rm A} = \text{const.}_{\rm A}$$
$$\log N + \log \mu_{\rm B} - \log \Gamma_{\rm B} = \text{const.}_{\rm B}$$

$\Gamma_{\rm A}$, $\Gamma_{\rm B}$ ... Lorentz factors of fluids A and B

+ Einstein equations for N, N^φ, A, B

+ Equation of state $E = E(n_{\rm A}, n_{\rm B}, \Delta^2)$

☞ generalize the elliptical spectral method solver (LORENE) by S.Bonazzola, J.-A. Marck, E.Gourgoulhon, J.Novak to the case of two fluids in order to solve for the stationary configuration

IV. Outlook

❏ Stationary configuration provides non–perturbed "background":

☞ analysis of oscillation modes of a two–fluid NS with $\Omega_A \neq \Omega_B$

[with N.Andersson, G.Comer]

☞ doubling of r-modes? frequencies? damping? mutual friction?

❏ Doubling of f- and p- modes:

G.Comer,D.Langlois,L.M.Lin,PRD **60**,104025(1999)

two–fluid NS model but no rotation and no entrainment

☞ Fig. 1

❏ Mutual friction damping of r–modes:

L.Lindblom, G.Mendell, PRD **61**, 104003 (2000)

with entrainment, but both fluids are co–rotating, ie. $\Omega_A = \Omega_B$

"resonant" damping as a function of entrainment ε!

☞ Fig. 2

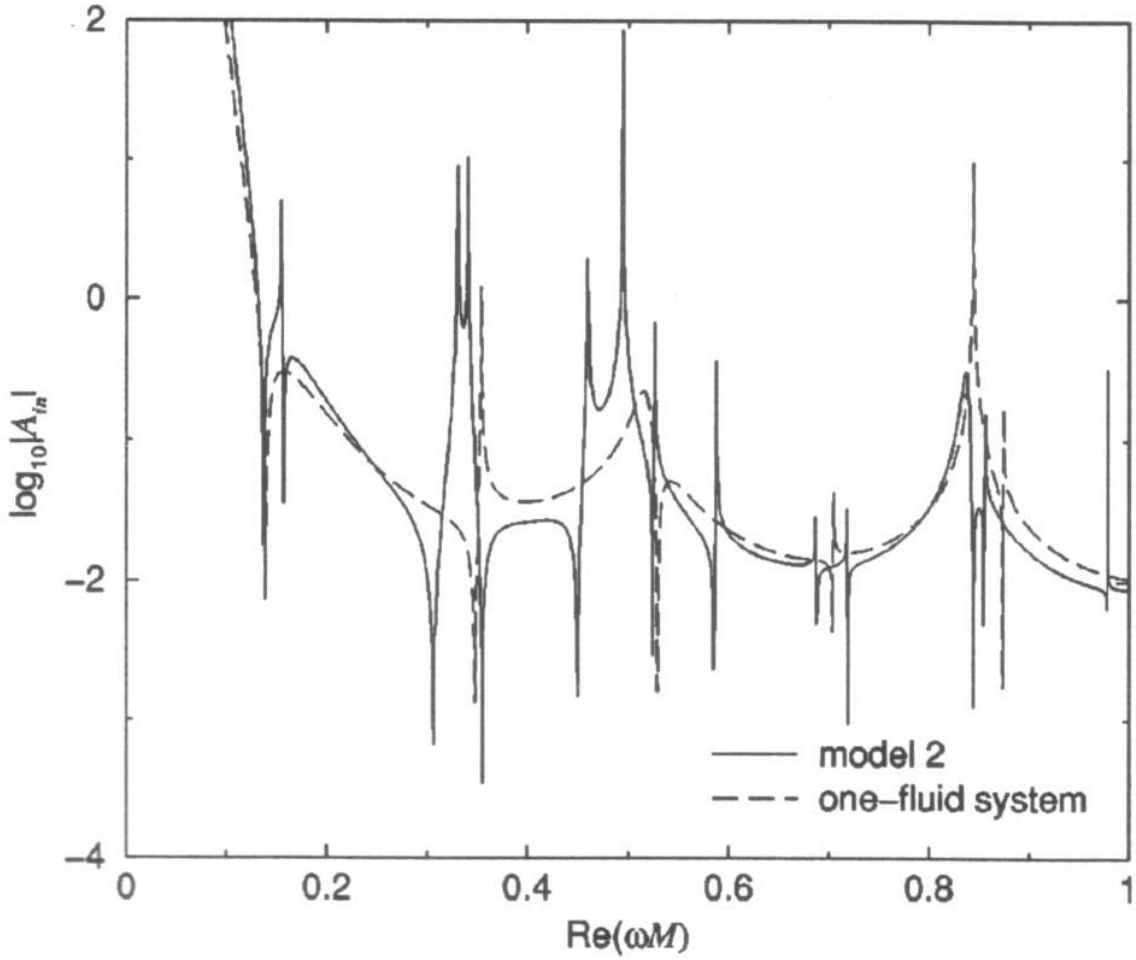


Fig. 1: Doubling of f- and p- modes

G.Comer,D.Langlois,L.M.Lin,PRD **60**,104025(1999)

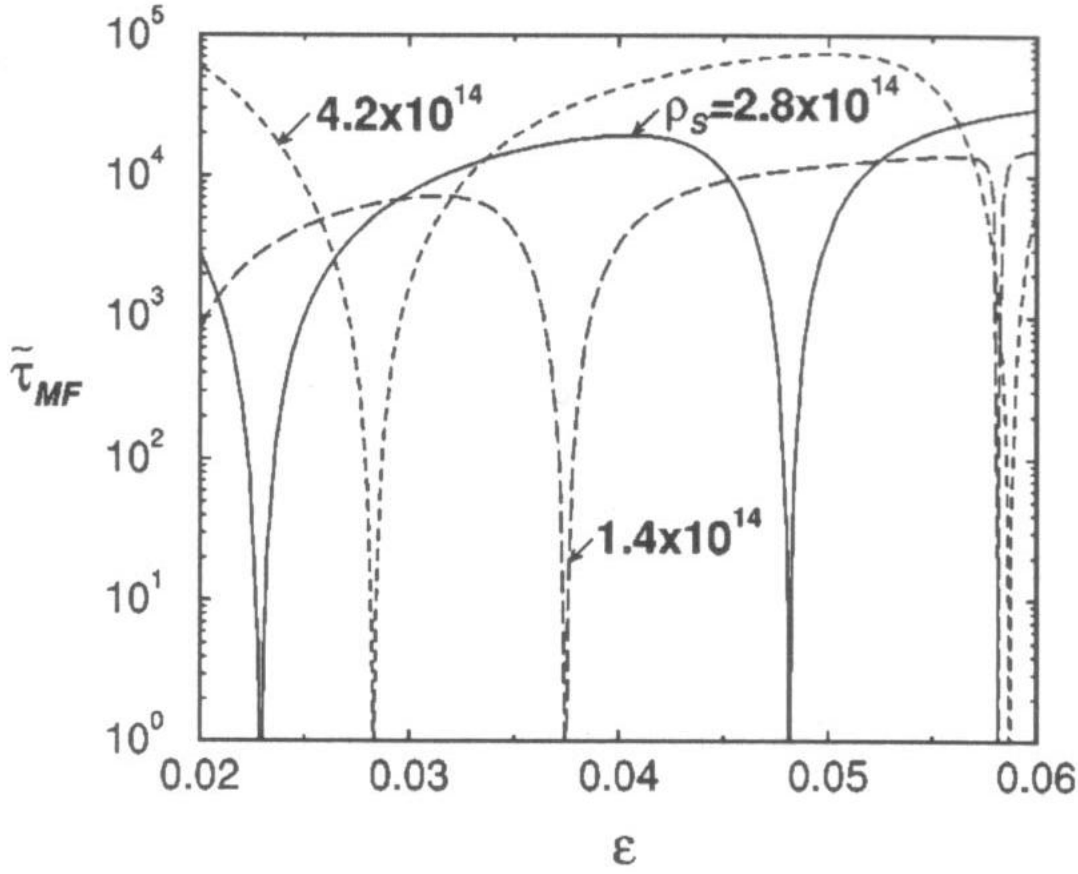


Fig. 2: "Resonant" Mutual friction damping of r–modes

L.Lindblom, G.Mendell, PRD **61**, 104003 (2000)