

2<sup>nd</sup> EU–Network meeting

7th–10th June 2001

Kallithea, Greece

## On the two–fluid neutron star model

### Plan

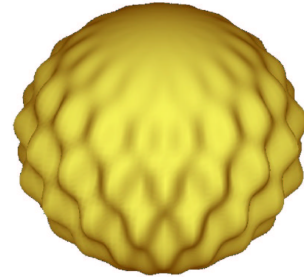
- I. Motivation of the two–fluid model
- II. Two–fluid hydrodynamics
- III. The Stationary Two–fluid NS model
- IV. Outlook

# I. Motivation

Neutron star oscillation modes

→ Gravitational wave (GW) emission

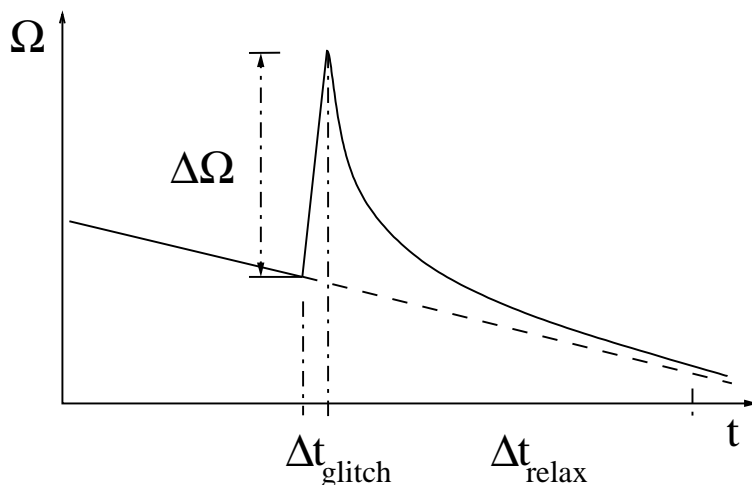
→ GW driven instabilities: eg. r-modes



N.Andersson; J.Friedman, S.Morsink (1998); ...

! most oscillation mode studies: single perfect fluid (Newtonian)

BUT: real pulsars show various rotation irregularities, eg. “Glitches”:



$$\Delta\Omega/\Omega \sim 10^{-6} - 10^{-9}$$

$$\Delta t_{\text{glitch}} \sim \text{minutes}$$

$$\Delta t_{\text{relax}} \sim \text{months}$$

internal “reservoir” of angular momentum: **very weakly coupled**

→ superfluid?  $\Omega_S > \Omega$  as  $\Omega \downarrow$

instability:  $\Omega_S \rightarrow \Omega \implies$  Glitch

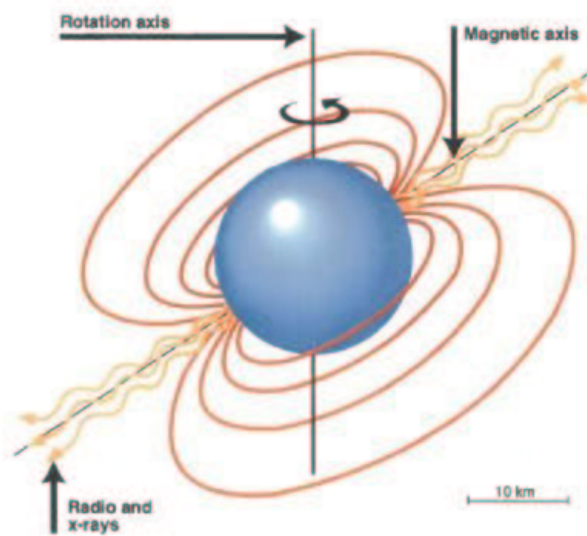
Observational motivation for

☞ SUPERFLUID

☞ Two-fluid NS model:  $\Omega_S \neq \Omega$

## “Astro–seismology” of neutron stars?

! detailed understanding and modeling of the **internal NS dynamics** is necessary, even prior to detection of oscillations!  
(contrary to helio–seismology)



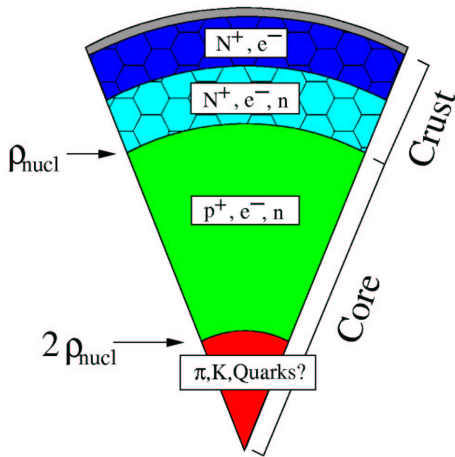
some necessary ingredients:

- General Relativity ( $\sim 40\%$  corrections)
- magnetic field ( $B \sim 10^{12}$  G)
- solid crust (viscous boundary layer?)
- **superfluidity + superconductivity**

☞ vortices

☞ two (or more) fluids

## Nuclear Physics: Internal structure



extremely degenerate matter:

$$T \lesssim 10^8 \text{ K} \ll T_F \sim 10^{12} \text{ K}$$

! thermal effects negligible

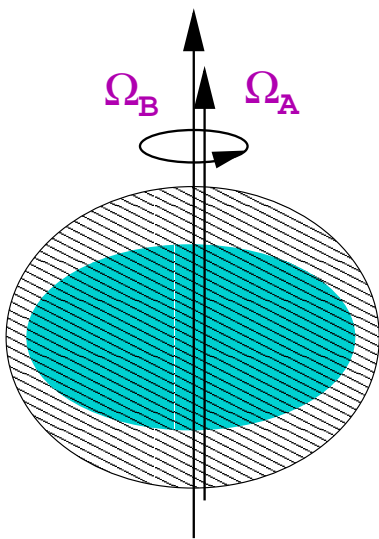
☞ superfluid neutrons

☞ superconducting protons

(transition temperature  $T_c \sim 10^{10} \text{ K}$ )

## Simplest non-trivial model: Two-fluid NS

- Fluid A: Superfluid neutrons (no viscosity!)
- Fluid B: all charged components (protons, electrons, muons, ...), coupled by magnetic field and viscosity



☐ can have  $\Omega_A \neq \Omega_B$

☐ base of all current glitch-models

☐ fluids A and B strongly coupled!

☞ “entrainment”

## II. Two-fluid hydrodynamics

B.Carter: covariant canonical hydrodynamics

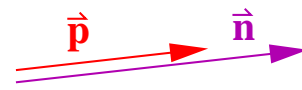
particle current:  $\vec{n}$                       particle density:  $n = \sqrt{-\vec{n}^2}$

particle conservation:  $\nabla_\alpha n^\alpha = 0$

energy density:  $E$                       momentum/particle:  $p_\alpha \equiv -\partial E / \partial n^\alpha$

equation of motion:  $n^\alpha \nabla_{[\alpha} p_{\beta]} = 0$

□ one fluid case: barotropic equation of state  $E = E(n)$

$\Rightarrow \vec{p} \parallel \vec{n}$                                             ...leads to usual hydro

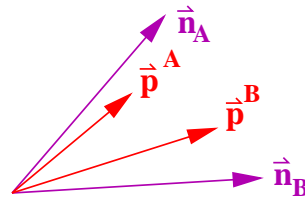
□ two fluids A, B:

conserved currents:  $\vec{n}_A, \vec{n}_B$ ,                      relative velocity:  $\Delta$

particle momenta:  $\vec{p}^A, \vec{p}^B$ ,                       $p_\alpha^A \equiv -\partial E / \partial n_A^\alpha$ ,                       $p_\alpha^B \equiv -\partial E / \partial n_B^\alpha$ ,

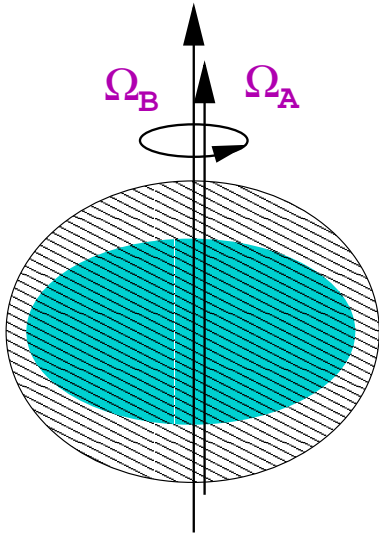
Equation of state:  $E = E(n_A, n_B, \Delta^2)$

$$\Rightarrow \begin{aligned} \vec{p}^A &= K^{AA} \vec{n}_A + K^{AB} \vec{n}_B \\ \vec{p}^B &= K^{BA} \vec{n}_A + K^{BB} \vec{n}_B \end{aligned}$$



! "ENTRAINMENT" ... consequence of the interaction  $A \leftrightarrow B$

# III. Stationary Two-fluid NS model



uniform rotations  $\Omega_A \neq \Omega_B$

Coupling  $A \leftrightarrow B$  by gravitational field and by entrainment!

previous projects: slow-rotation expansion

R.Prix, A&A **352**,623 (1999) (Newtonian)

N.Andersson, G.Comer, CQG **18**, 969 (2001) (GR)

→ entrainment not fully included!

## (A) Newtonian slow rotation expansion WITH entrainment

R.Prix, G.Comer, N.Andersson, in preparation

Two equations of motion + stationarity + axisymmetry:

⇒ Two integrals of motion:

$$\mu_A + m_A \Phi - \frac{1}{2} m_A v_A^2 = \text{const}_A$$

$$\mu_B + m_B \Phi - \frac{1}{2} m_B v_B^2 = \text{const}_B$$

+ Poisson equation:  $\nabla^2 \Phi = 4\pi G \rho$

**Slow Rotation:** Taylor expansion in  $\Omega_A$  and  $\Omega_B$  up to  $\mathcal{O}(\Omega^2)$ :

$$\text{eg. } \Phi(r, \theta) = \Phi^{(0)}(r) + \underbrace{\delta\Phi(r, \theta)}_{\mathcal{O}(\Omega^2)} + \mathcal{O}(\Omega^4)$$

⇒ numerical solution (ODE) for any given EOS:  $E = E(n_A, n_B, \Delta^2)$

! analytic solution  $\mathcal{O}(\Omega^2)$  for a particular class of EOS:

$$E = \underbrace{(\kappa_A n_A^2 + \kappa_B n_B^2)}_{\text{sum of two } \gamma = 2 \text{ polytropes!}} + 2\sigma n_A n_B + \underbrace{\beta n \Delta^2}_{\text{entrainment}}$$

How good is this entrainment model?

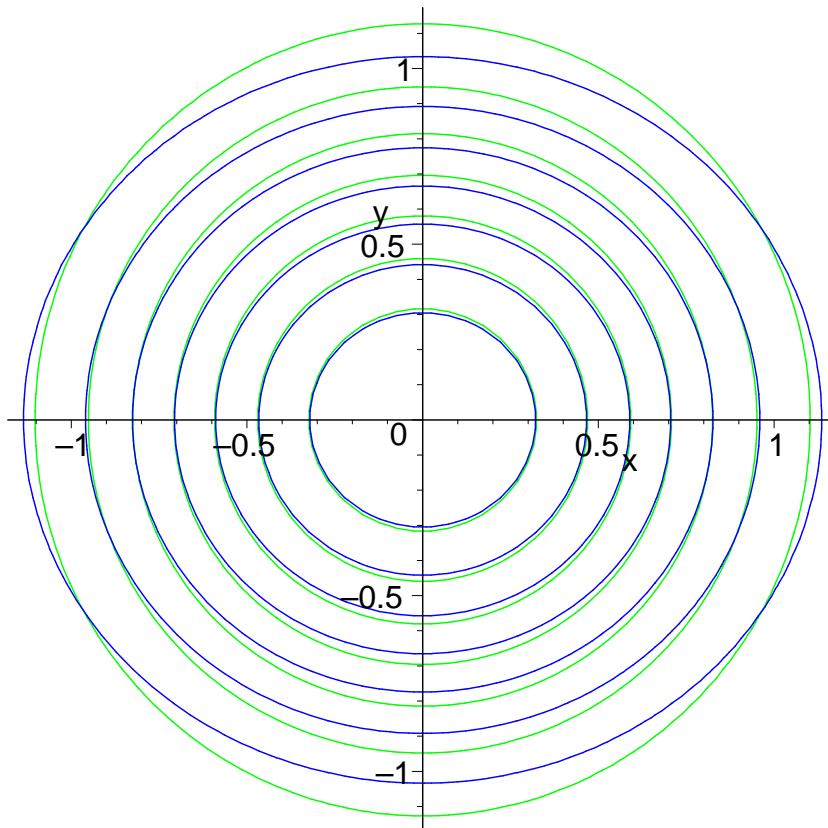
equivalence “entrainment”  $\longleftrightarrow$  “effective masses”  $m^*$ :

$$! \beta = \text{const.} \quad \iff \quad m_p^* = \text{const.} !$$

nuclear physics:  $m_p^* \approx 0.6 m_p$  and slowly varying with density!

### Some plots of the analytic solution:

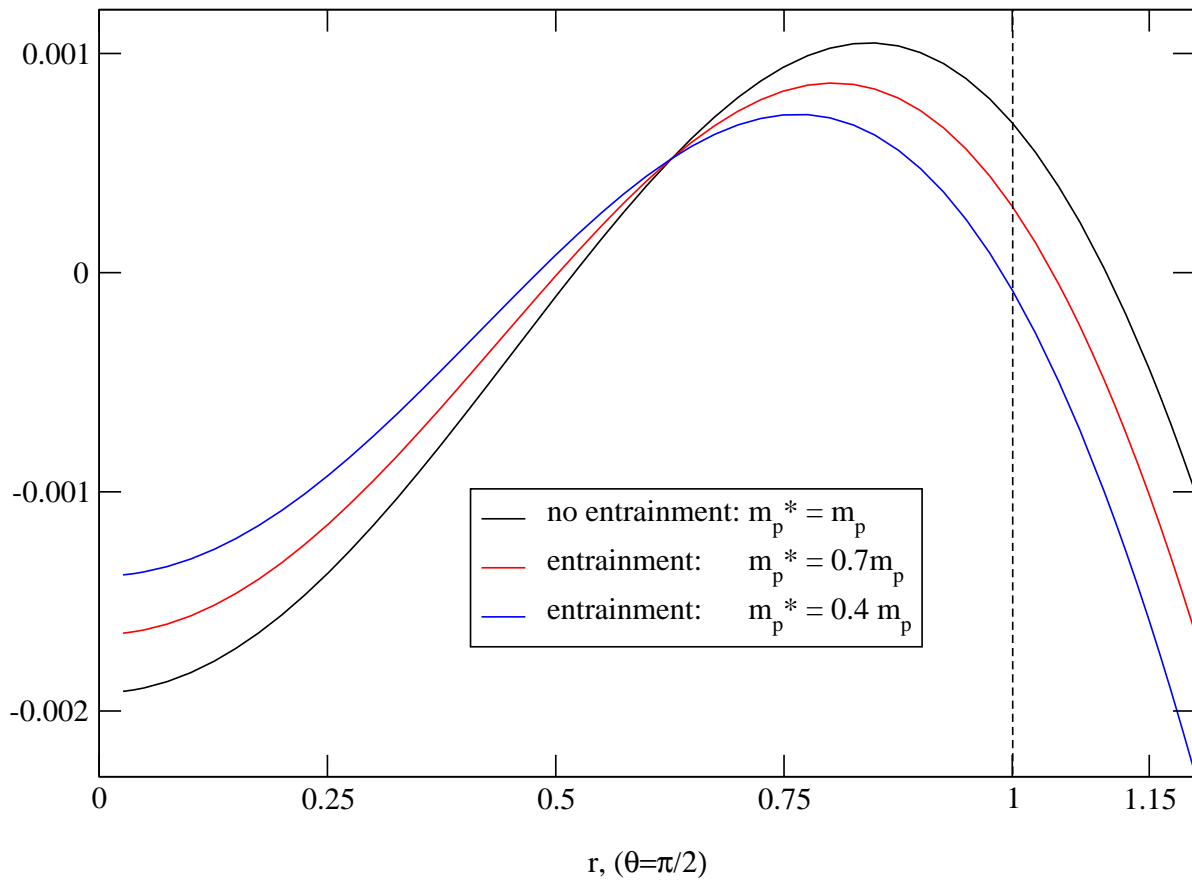
isodensity surfaces of  $n_A$  and  $n_B$  ( $\Omega_A \approx 4000 \text{ s}^{-1}$ ,  $\Omega_B \approx 0.5 \Omega_A$ ):



$$n_p(r, \theta) = n_p^{(0)}(r) + \delta n_p(r, \theta) + \mathcal{O}(\Omega^4)$$

proton density correction  $\delta n_p(r, \theta = \pi/2)$

in equatorial direction





## (B) “Exact” numerical solution in GR

current project with J.Novak (Meudon) and G.Comer

2 Killing vectors: stationarity ...  $\vec{t}$       axisymmetry ...  $\vec{\varphi}$

most general metric in MSQI gauge: ( $N$  ... lapse,  $N^\varphi$  ... shift)

$$ds^2 = -(N^2 - N_\varphi N^\varphi) dt^2 - 2N_\varphi d\varphi dt + A^2 (dr^2 + r^2 d\theta^2) + B^2 r^2 \sin^2 \theta d\varphi^2$$

symmetries  $\vec{t}, \vec{\varphi} \implies$  two first integrals:

$$\log N + \log \mu_A - \log \Gamma_A = \text{const.}_A$$

$$\log N + \log \mu_B - \log \Gamma_B = \text{const.}_B$$

$\Gamma_A, \Gamma_B$  ... Lorentz factors of fluids A and B

+ Einstein equations for  $N, N^\varphi, A, B$

+ Equation of state  $E = E(n_A, n_B, \Delta^2)$

☞ generalize the elliptical spectral method solver (LORENE) by S.Bonazzola, J.-A. Marck, E.Gourgoulhon, J.Novak to the case of two fluids in order to solve for the stationary configuration

## IV. Outlook

□ Stationary configuration provides non-perturbed “background”:

☞ analysis of oscillation modes of a two-fluid NS with  $\Omega_A \neq \Omega_B$

[with N.Andersson, G.Comer]

☞ doubling of r-modes? frequencies? damping? mutual friction?

□ Doubling of f- and p- modes:

G.Comer,D.Langlois,L.M.Lin,PRD **60**,104025(1999)

two-fluid NS model but **no** rotation and **no** entrainment

☞ Fig. 1

□ Mutual friction damping of r-modes:

L.Lindblom, G.Mendell, PRD **61**, 104003 (2000)

**with** entrainment, but both fluids are co-rotating, ie.  $\Omega_A = \Omega_B$

“resonant” damping as a function of entrainment  $\varepsilon$ !

☞ Fig. 2

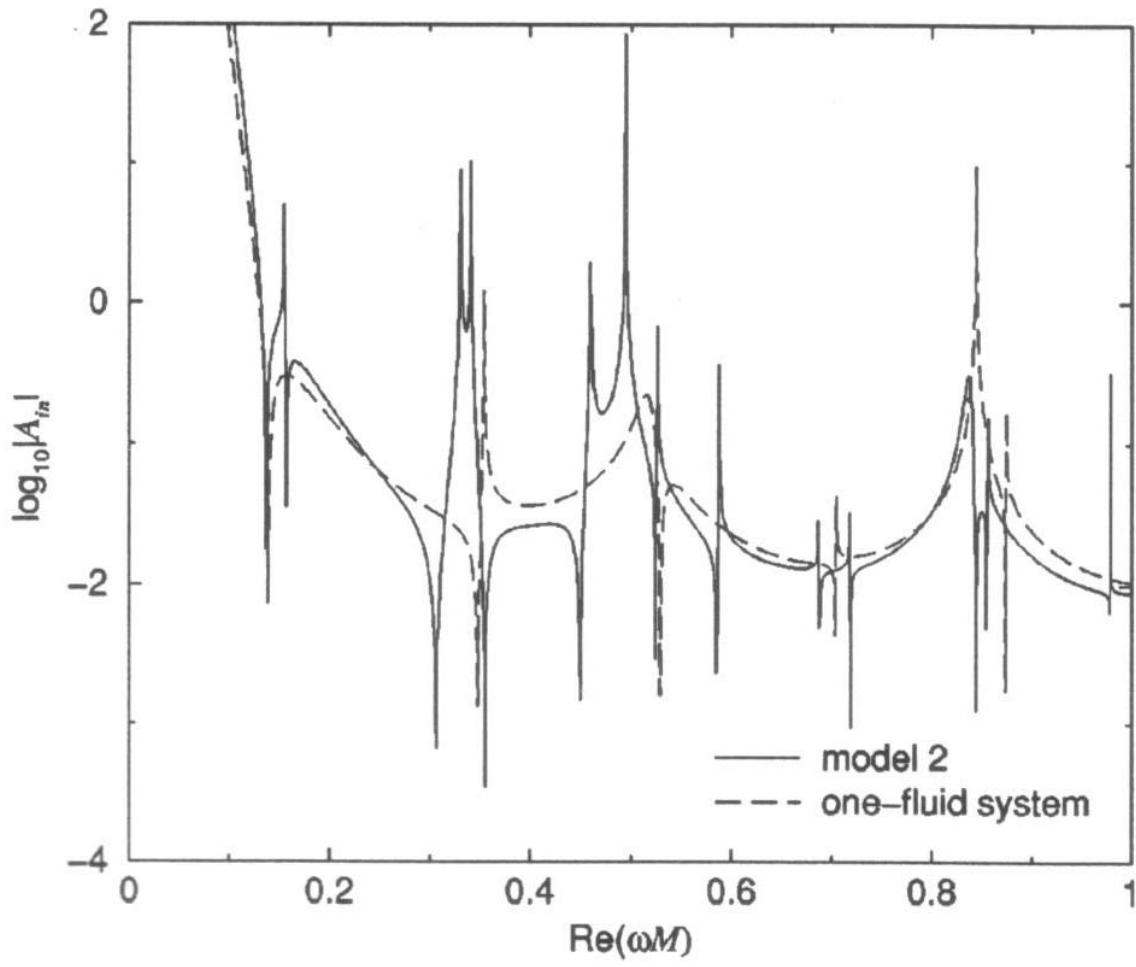


Fig. 1: Doubling of f- and p- modes

G.Comer,D.Langlois,L.M.Lin,PRD **60**,104025(1999)

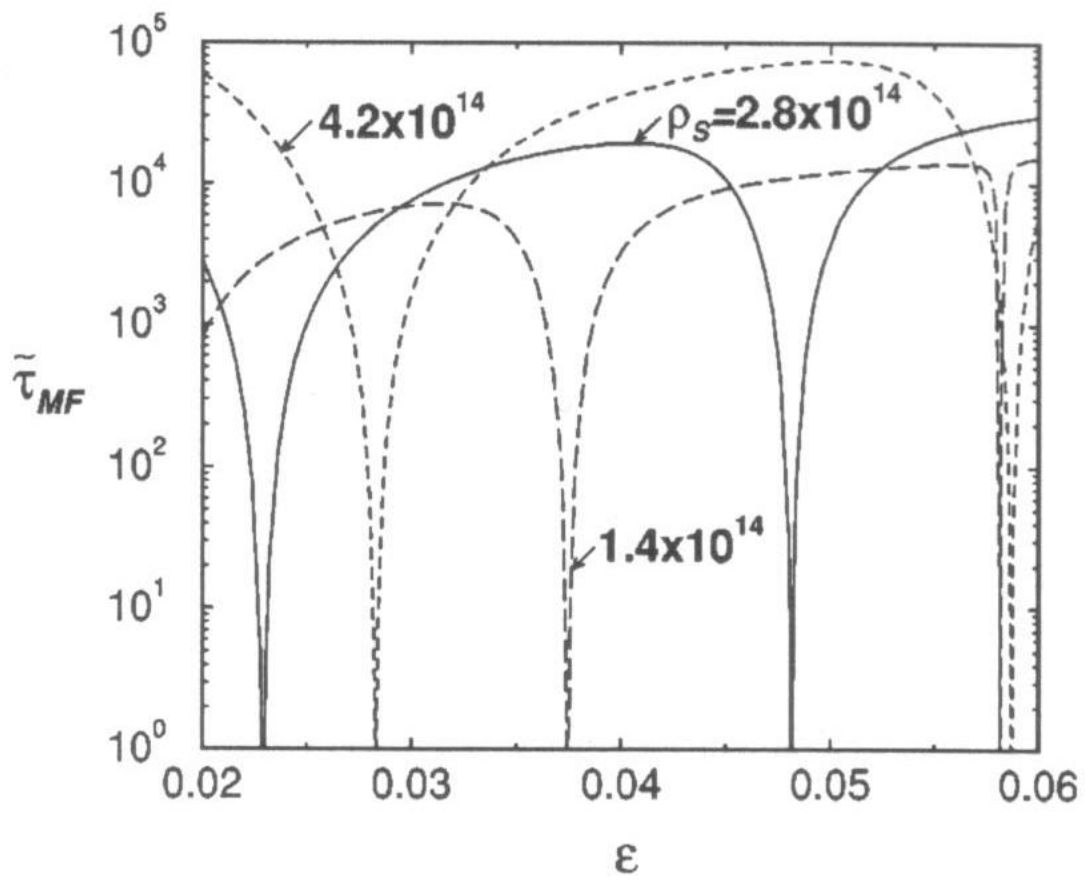


Fig. 2: “Resonant” Mutual friction damping of r-modes

L.Lindblom, G.Mendell, PRD **61**, 104003 (2000)